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The Nuclear Excitation of Krypton and Rhodium

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Kr^{83*} has been produced by the direct x-ray bombardment of the element as well as from Kr^{82} by the $n-\gamma$ process. A study of the nuclear spectrum of Rh^{103} by x-ray excitation gave values of 1.26±, 0.03 Mev, 1.64, 2.02, 2.37, 2.71, and 3.05 for the higher nuclear states which combine with the metastable level. Measurements of the absorption of the conversion electrons in gas indicate that the energy of the metastable state is 40 Kev about the ground level. It was found that Rh^{103*} can also be produced by the direct bombardment of rhodium with electrons.

EXCITATION OF Kr^{83}

LANGSDORF and Segrè[1] observed a 113-minute period in krypton grown in the reaction

$$Se^{83}\xrightarrow{\beta}Br^{83}\xrightarrow{\beta}Kr^{83*}\xrightarrow{\gamma}Kr^{83},$$

while investigating the products of thorium and uranium fission.

The γ-rays emitted in the transition from

$$Kr^{83*}\xrightarrow{\gamma}Kr^{83}$$

have been found to be 0.029 and 0.046 Mev.[2]

Kr^{83*} was later produced[3,4] in the Se-α-n and Kr-d-p reactions.

Counters with brass cathodes of 2.5-cm diameter have been constructed and filled with a mixture of krypton gas to a pressure of 20 cm of Hg and ethyl ether to a pressure of 2 cm of Hg. These counters were found to react much like the usual argon-ether counters. After bombarding such a counter with intense x-rays of maximum energy 2.5 Mev, a moderate activity could be observed which decayed with a half-life of about 115 minutes. This activity was produced by the γ-rays and conversion electrons emitted in the decay of krypton.^{83*}

Material with this same half-life period was also produced by bombarding krypton gas with slow neutrons produced from the Be-γ-n process. The neutrons were slowed down by allowing them to pass through paraffin. Since neither the n-$2n$ nor the n-n process can take place at these low energies, the activity must have been produced by the reaction $Kr^{82}+n\rightarrow Kr^{83*}+\gamma$.

EXCITATION OF Rh^{103}

The presence of a metastable level in stable rhodium^{103} was first indicated by Flammersfeld[5] who induced an activity of half-life 48±5 minutes by bombarding this element with 3 Mev neutrons. The process was assumed to be a Rh-n-n reaction. Only very soft γ-rays were observed. Lead of thickness 0.3 g/cm^2 was found

[1] A. Langsdorf, Jr., and E. Segrè, Phys. Rev. **57**, 105 (1940).
[2] A. C. Helmholz, Phys. Rev. **60**, 415 (1941).
[3] E. P. Clancy, Phys. Rev. **58**, 88 (1940).
[4] E. P. Clancy, Phys. Rev. **60**, 87 (1941).
[5] A. Flammersfeld, Naturwiss. **32**, 36 (1944).

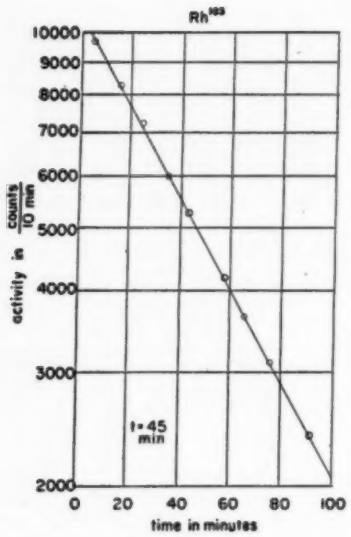

FIG. 1. Decay curve of Rh^{103*} to Rh^{103}. The half-life indicated is 45±1 minute.

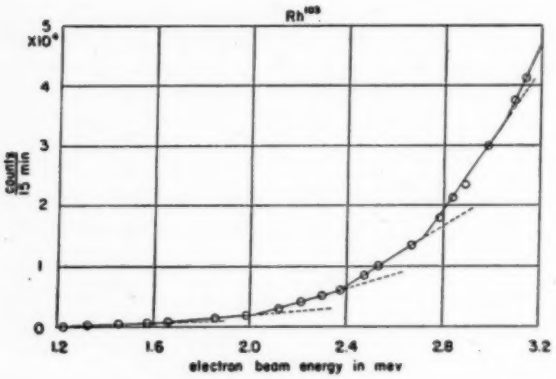

FIG. 2. X-ray excitation curve for Rh^{103}. The activity was produced by bombarding the samples for 10 minutes using an electron beam current of 100 microamperes.

to absorb all of the radiation while 0.85 g/cm² of graphite reduced the intensity to 60 percent.

We have previously reported[6] the production of Rh^{103*} by both x-ray and electron bombardment of elementary rhodium.

Although rhodium has been used in these laboratories for some time as a neutron detector in the presence of strong x-rays, it was not until the rhodium samples were placed directly in the active counting region that this x-ray induced activity could be observed.

A decay curve, Fig. 1, has been obtained by following the activity induced in a rhodium cathode counter through 100 minutes. This curve indicates that the half-life period is 45±1 minutes which is in good agreement with the data of Flammersfeld.

The energy of the conversion electrons produced in the transition from Rh^{103*} to the ground state has been measured by absorption in argon gas. This value plus the binding energy of the K electrons in rhodium yields a value of 0.040 Mev for the energy of the metastable level.

An x-ray excitation curve has been obtained in a manner described elsewhere.[7,8] This is shown in Fig. 2. The breaks in the curve, indicating the positions of the higher nuclear energy levels which combine with the metastable state, were found at 1.26±, 0.03 Mev, 1.64, 2.02, 2.37, 2.71, and 3.05 Mev.

TABLE I. Collection of combining nuclear levels.

Element	Half-Life	Energy (Mev)	ΔE (Mev)
Rh^{103}	45 min.	0.04 (metastable)	
		1.26 (threshold)	(1.22)
		1.64	0.38
		2.02	0.38
		2.37	0.35
		2.71	0.34
		3.05	0.34
		Average	0.36
$Ag^{107 \text{ or } 109}$	40.4 sec.	0.093 (metastable)	
		1.18 (threshold)	(1.09)
		1.59	0.41
		1.95	0.36
		2.32	0.37
		2.76	0.44
		3.13	0.37
		Average	0.39
Cd	48.8 min.	0.20 (metastable)	
		1.25 (threshold)	(1.05)
		1.68	0.43
		2.08	0.40
		2.56	0.48
		Average	0.44
In^{115}	4.42 hours	0.338 (metastable)	
		1.12 (threshold)	(0.78)
		1.55	0.43
		2.13	0.58
		2.63	0.50
		Average	0.50
Au^{197}	7.5 sec.	0.25 (metastable)	
		1.22 (threshold)	(0.97)
		1.68	0.46
		2.15	0.47
		2.56	0.41
		2.97	0.41
		Average	0.44

[6] M. L. Wiedenbeck, Phys. Rev. 67, 267 (1945).

[7] M. L. Wiedenbeck, Phys. Rev. 67, 92 (1945).

[8] M. L. Wiedenbeck, Phys. Rev. 68, 1 (1945).

From these data a nuclear energy level diagram can be drawn, Fig. 3, showing the various nuclear transitions which occur.

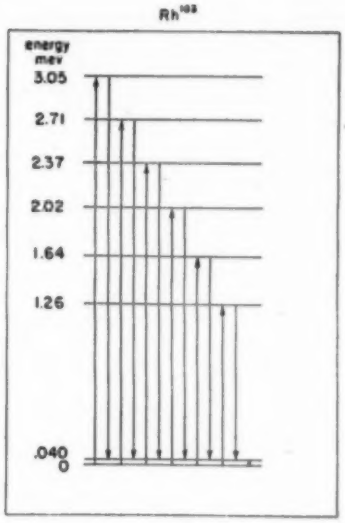

FIG. 3. Nuclear energy level diagram for Rh^{103} showing transitions which occur.

DISCUSSION

A summary of the nuclear energy levels of stable nuclei which have been studied by x-ray excitation is given in Table I.

The error incurred in determining the intersection of the straight line portions of the thick target x-ray excitation curve does not exceed ± 0.03 Mev. Such lines are determined from numerous points obtained under identical geometrical conditions. In any given bombardment, the energy of the electron beam can be held constant to a value of ± 0.02 Mev and the current does not vary by more than 1 percent. The maximum error in the difference between two adjacent levels is therefore ± 0.06 Mev. It may be noted that with few exceptions the levels in any given element thus far studied fall within this limit. The average spacing varies, however, and increases regularly for rhodium, silver, cadmium, and indium.[9]

[9] B. Waldman and M. L. Wiedenbeck, Bull. Am. Phys. Soc. 17, No. 5 (1942).

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Collision Cross Sections for 25-Mev Neutrons

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Collision cross sections of a number of elements for high energy neutrons have been measured. Neutrons with a maximum energy of 25.4 Mev were obtained by bombarding lithium with 10.2 Mev deuterons. The reaction $C^{12}(n, 2n)C^{11}$, which has a measured threshold energy of approximately 21 Mev, was used as an energy sensitive detector for the transmission measurements. The cross section obtained for the neutron-proton collision process was $0.39 \pm 0.03 \times 10^{-24}$ cm². This is higher than the cross section calculated for s-scattering (0.35×10^{-24} cm²), but agrees well with the value of 0.40×10^{-24} cm² predicted by the symmetrical meson theory of Rarita and Schwinger. Measurements on other nuclei ranging from carbon to mercury show that the collision radius is given by $R' = b + r_0 A^{\frac{1}{3}}$, with $b = 1.7 \pm 0.4 \times 10^{-13}$ cm and $r_0 = 1.22 \pm 0.15 \times 10^{-13}$ cm. These measurements are in good agreement with the inelastic cross-section measurements of Grahame and Seaborg. The value of r_0 is somewhat lower than the values deduced from p-n reactions, Coulomb energies, and α-particle decay.

INTRODUCTION

KITTEL and Breit[1] have calculated the dependence of the proton-neutron collision cross section on the energy of the incident neutrons. They have shown that the experimentally determined cross sections up to an energy[2] of 15 Mev can be explained on the basis of s-wave scattering. The magnitude of p wave scattering is small at 15 Mev, and is masked by the spherical scattering. The present experiment was undertaken to extend the measurements of the collision cross section to 25 Mev. At this energy, the s-wave scattering is reduced to 50 percent of its value at 15 Mev, while the p-wave scattering is increased.

High energy neutrons can also be used to test current theories of nuclear radii. It is generally assumed that the nuclear radius R is given by

$$R = r_0 A^{\frac{1}{3}}, \qquad (1)$$

where A is the atomic mass, and r_0 is a constant. At present, the experimental evidence is in approximate agreement with Eq. (1), although the values of r_0 deduced from excitation functions, α-particle decay, and Coulomb energies range[3-5] from 1.3 to 2.2×10^{-13} cm. These experiments are generally confined to small regions of the periodic table and the radii are obtained indirectly. Scattering experiments with neutrons can be performed with elements selected from a major part of the periodic table. Transmission data yield the nuclear radii directly, since the results are not obscured by Coulomb scattering.

The geometrical radius R will be obtained from collision experiments if the neutron energy is sufficiently high. At 3 Mev strong resonances have been observed.[6] However, at higher energies, the compound nucleus will have an excitation energy in the region where the width of levels is large compared with their separation. Resonance effects will be minimized and the inelastic scattering cross section[3,7] will be πR^2. This condition will be reached when $\lambda = \lambda/2\pi$ is much smaller than the nuclear radius, where λ is the wave-length of the incident neutron. For the light nuclei, the neutron energy should be $\gg 8$ Mev.

Since the high energy neutron sources now available have a heterogeneous spectrum, it is necessary to discriminate against the low energy fraction by using a detector insensitive to these neutrons. At present, the only experiments which satisfy this condition are those of Salant and Ramsey,[2] and Graham and Seaborg.[8] The

* Now at Los Alamos, New Mexico.
[1] C. Kittel and G. Breit, Phys. Rev. **56**, 744 (1939).
[2] E. O. Salant and N. F. Ramsey, Phys. Rev. **57**, 1075A (1940).
[3] V. F. Weisskopf and D. H. Ewing, Phys. Rev. **57**, 472 (1940).
[4] H. A. Bethe, Rev. Mod. Phys. **9**, 166 (1937).
[5] R. D. Present, Phys. Rev. **60**, 28 (1941).
[6] See for example, W. H. Zinn, S. Seeley, and V. W. Cohen, Phys. Rev. **56**, 260 (1939).
[7] H. A. Bethe, Phys. Rev. **57**, 1125 (1940).
[8] D. C. Grahame and G. T. Seaborg, Phys. Rev. **53**, 795 (1938).

former used the reaction $Cu^{63}(n, 2n)Cu^{62}$ with a threshold at 12 Mev, but measured only the neutron-proton and neutron-carbon cross sections at 14–15 Mev. Graham and Seaborg measured the inelastic scattering cross sections for a number of nuclei by using (Ra+Be) neutrons with maximum energy of 13.7 Mev. They used the reaction $Fe^{56}(n, p)Mn^{56}$ to give a radioactive detector insensitive to neutron energies below 7 Mev. Their results will be discussed below.

In a transmission type of experiment, neutrons are removed from the incident beam by elastic as well as inelastic collisions. Since the geometry one uses is necessarily finite, correction must be made for neutrons scattered into the detector. Inelastic collisions most probably result in the emission of neutrons of considerably reduced energy[9] but if the detector threshold is sufficiently high, the contribution of these neutrons to the measured intensity is negligible. The elastic scattering at high energy is essentially the diffraction of the neutron beam by the absorbing nuclei. The cross section for this process is πR^2, so that the total cross section (elastic plus inelastic) for removal of neutrons is $2\pi R^2$.

The geometrical corrections can be calculated by using the differential cross section given by Placzek and Bethe.[10]

$$\sigma(\varphi) = R^2[J_1(\kappa R\varphi)/\varphi]^2, \qquad (2)$$

where J_1 is a Bessel function of the first kind, φ is the angle of scattering, and κ is the wave number of the incident neutrons. Frankel[11] and Present[8] have shown that Eq. (2) describes satisfactorily the angular distribution of elastically scattered neutrons observed by Wakatuki and his co-workers.[12] Comparison of the scattering data for different elements indicates that the value of R to be used is not that defined by Eq. (1), but rather

$$R' = b + r_0 A^{\frac{1}{3}}, \qquad (3)$$

where b is a constant.

Bethe[7] and Present[8] have suggested that the formula for the neutron collision radius should be of this form. Bethe assumes that the nucleus is an opaque sphere of radius $r_0 A^{\frac{1}{3}}$. However, the "screening" radius for neutrons is increased above this geometrical radius by a quantity b representing the finite range of nuclear forces. This quantity is constant for all nuclei and is $\sim 10^{-13}$ cm. Present discusses the problem of nuclear structure in greater detail. He suggests that the nucleus has a core of constant density and an outer shell of diminishing density of thickness $\sim 10^{-13}$ cm. The latter is independent of atomic weight (for $A > 50$). The constant quantity b is therefore increased to $\sim 2 \times 10^{-13}$ cm. This value is in agreement with the elastic scattering data mentioned above.

EXPERIMENTAL METHOD

Neutrons for the present experiment were obtained by bombarding lithium with 10.2 ± 0.3 Mev deuterons produced by the Harvard cyclotron. The mass values of Allison[13a] give an energy release of 15.2 Mev in the reaction $Li^7 + D^2 \rightarrow Be^8 + n^1$. Cloud chamber experiments by Stephens[14] yielded a neutron distribution which extends from low energies to a maximum corresponding to a Q of 15.1 Mev, with a pronounced group of neutrons corresponding to a Q of 11.8 Mev. The maximum energy of the neutrons for a deuteron energy of 10.2 Mev is calculated to be 25.4 Mev and 22.1 Mev for the two neutron groups.

Carbon was used as an energy-sensitive neutron detector. The reaction $C^{12}(n, 2n)C^{11}$ yields the 20.5-min. positron activity of C^{11}. The threshold calculated from the mass values of Mattauch, Haxby, and Barkas[13b, c, d] is 20.4 Mev. An approximate check on the threshold was obtained experimentally by placing carbon at various angles with respect to the forward direction of the neutron beam. The maximum energy corresponding to each angle of emission of the neutrons from the thick lithium target was calculated. The results are shown in Fig. 1 where the relative activity is plotted against the angle

[9] V. F. Weisskopf, Phys. Rev. **52**, 295 (1937).
[10] G. Placzek and H. A. Bethe, Phys. Rev. **57**, 1075A (1940).
[11] S. Frankel, Phys. Rev. **59**, 216A (1941).
[12] T. Wakatuki, Proc. Phys.-Math. Soc. Japan **22**, 430 (1940).

[13] (a) Allison, Miller, Perlow, Skaggs, and Smith, Phys. Rev. **58**, 178 (1939). (*b*) J. Mattauch, Phys. Rev. **57**, 1155 (1940). (c) Haxby, Shoupp, Stephens, and Wells, Phys Rev. **58**, 1035 (1940). (d) W. H. Barkas, Phys. Rev. **55**, 691 (1939).
[14] W. E. Stephens, Phys. Rev. **53**, 224 (1938).

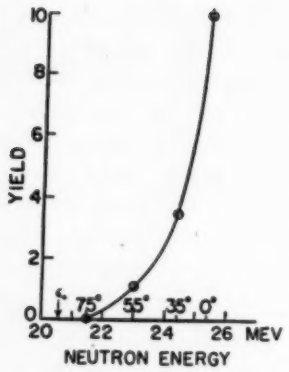

FIG. 1. Excitation function for the reaction $C^{12}(n, 2n)C^{11}$.

of emission and the corresponding maximum energy. At 75° the intensity was less than 1 percent of the 0° activity, indicating that the threshold for the n-$2n$ reaction is in the neighborhood of 21.5 Mev, in agreement with the calculated value. This excitation curve indicates that the effective energy of the neutrons lies between 24 and 25 Mev. The high gradient of the excitation function indicated that little difficulty would be experienced with neutrons scattered by the cyclotron. The massive parts of the latter which might reflect neutrons into the detector either were several feet away or could reflect only neutrons of reduced energy.

The experimental arrangement for the transmission experiments is shown in Fig. 2. The effective area of the lithium target was 2.0×0.3 cm². The target was prepared by pounding metallic lithium into a cylindrical depression in a $\frac{1}{4}$-inch water-cooled brass plate. The thickness of the lithium was $\frac{1}{32}$ inch. This was sufficient to reduce the energy of the deuterons below 7 Mev, lower energies being useless for the present experiment. It was thin enough to eliminate the necessity of more elaborate cooling of the target for beam currents of 50 to 60μa.

The deuteron beam was monitored by a graphite disk placed 1 inch in front of the target. Both monitor and detector were irradiated simultaneously to avoid corrections for bombardment time and fluctuations in the beam intensity. The detector[15] was made by quartering a hollow graphite cylinder (4.5 cm long) parallel to its axis. During irradiation the four pieces were stacked together and placed longitudinally with respect to the neutron beam. The cross section of the dectector was 1.9×1.6 cm² and its center was 22.4 cm from the target. The scatterers had a cross section of 1 sq. in. and were placed midway between the target and detector. Detector and scatterers were supported by thin wooden cradles. Wooden clamps and braces were used to minimize the effect of scattered neutrons. Target, monitor, scatterer, and detector were partially aligned by a set of pins. Final adjustments were made visually, a procedure found to be sufficiently accurate.

After irradiation for 40 minutes with deuteron currents of 50–60 μa, the monitor and detector were removed. The radioactivity induced in the monitor was observed with a Lauritsen electroscope, while the detector was wrapped about a thin-walled Geiger counter which fed into a "scale of 32" recording unit. Both electroscope and counter were calibrated with a U_2O_3 standard several times during each run to correct for the small variations in the sensitivity of the counter. These rarely exceeded a few percent during the several hours of observation. Typical decay curves are shown in Fig. 3, where the backgrounds of 0.16 div/min. and 0.6 counts/sec. have been subtracted from the observed activities. Preliminary observations indicated a weak activity with a period of several hours in the detector, but this activity was eliminated by surrounding the detector with a thin layer (0.8 g/cm²) of Cd. It is seen that the decay curves are simple over the observable range. The half-periods varied between 20 and 21 min., in agreement with previously determined values.

RESULTS

The results are summarized in Table I. The first and second columns give the element and

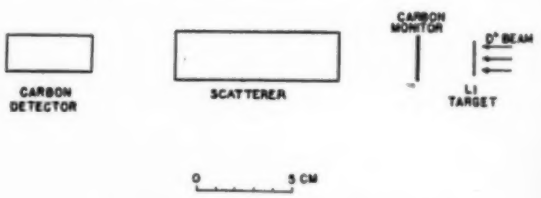

FIG. 2. Experimental arrangement for transmission measurements.

[15] The author is indebted to the National Carbon Company of Cleveland, Ohio, for providing him with the high grade artificial graphite necessary for this experiment.

the composition of the scatterer. The third column gives the thickness of scatterer in grams per cm². The fourth column gives the ratio of the activities of detector to monitor. These ratios are converted to percent transmission in the fifth column. The sixth column gives the corresponding values of the experimental cross section (in 10^{-24} cm²) obtained by use of Eq. (4) below. The last column gives the cross sections after the corrections described below were applied to the observed values. Only single measurements were made for Al, Cl, Ag, and Hg. Because of the consistency of the previous data and the fact that these runs satisfied the criteria adopted in judging a run satisfactory, these measurements have been listed in Table I. These criteria were: stable operation of the cyclotron; absence of damage to the Li film; and stability of the detecting devices. The errors of the measured cross sections (column 6) are estimated from the consistency of the data to be less than 8 percent for hydrogen, 5 percent for carbon, and 10 percent for the other elements. The values for hydrogen and carbon have been corrected for the chemical composition of the paraffin and the natural graphite used.

No correction has been made for neutrons scattered into the detector by the surroundings. Because of the weak intensity of the induced activity this effect could not be accurately determined by inverse square measurements. The small absorption cross sections precluded the possibility of absorbing out the direct neutrons. The detector showed no detectable activity when irradiated directly above and below the target. This indicated negligible contribution by those neutrons leaving the target in the forward direction and being scattered back by the magnet yoke, water tanks, etc. Neutrons leaving at larger angles had reduced energy and had to travel more than a foot before striking the cyclotron. These two factors and the small probability of large angle scattering make one reasonably sure that the effect of scattered neutrons could be neglected.

TABLE I. Summary of experimental results.

Element	Compound used	g/cm²	Ratio det./mon.	Percent trans.	$\sigma_{obs}\times10^{24}$ cm²	$\sigma_{corr}\times10^{24}$ cm²
No scatterer			2.06			
			2.05			
			2.06			
			2.06			
			2.06			
H	paraffin	5.35	1.31	63.6		
			1.39	67.5		
		7.39	1.09	52.9		
			1.13	54.9	0.39*	
			1.19	57.8		
			1.13	54.9		
			1.15	55.9		
C	C	8.70	1.32	64.0		
			1.34	65.0		
		13.1	1.09	52.9	1.07*	1.29
		17.4	0.84	40.7		
			0.80	38.8		
O	H_2O	7.82	1.22	59.3		
			1.19	57.8	1.27	1.60
			1.18	57.3		
check	$C_3H_4O_3$	8.20	1.22	59.2	6.35	
Al	Al	20.65	1.06	51.5	1.43	1.85
Cl	CCl_4	12.4	1.48	71.9	1.42	1.88
Cu	Cu	27.8	1.27	61.9	1.79	2.50
		39.2	1.06	51.5		
Ag	Ag	44.3	1.13	54.9	2.41	3.70
Hg	Hg	106	0.743	36.0	3.18	5.25

* Corrected for chemical composition.

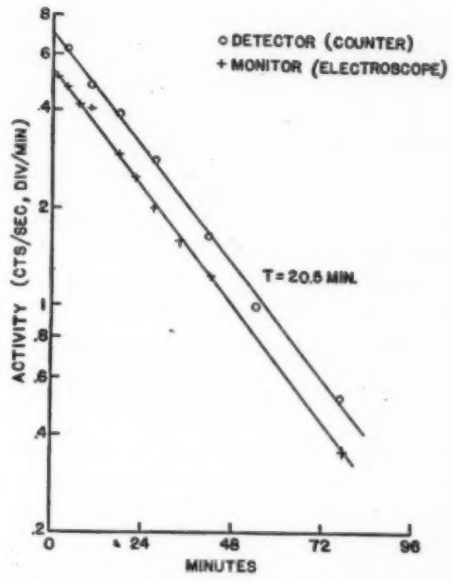

FIG. 3. Decay of C^{11} radioactivity induced in detector and monitor.

CALCULATION OF CORRECTED CROSS SECTIONS

If the geometry were ideal, the neutron intensity would vary exponentially with absorber thickness:

$$I = I_0 \exp\left[-N(\sigma_i+\sigma_e)x\right], \qquad (4)$$

where N is the number of absorbing nuclei per cm², σ_i and σ_e are the inelastic and elastic scattering cross sections, and x is the scatterer thickness in cm. Because of the finite size of the source, and scatterer and detector, correction must be made for obliquity of neutron paths and

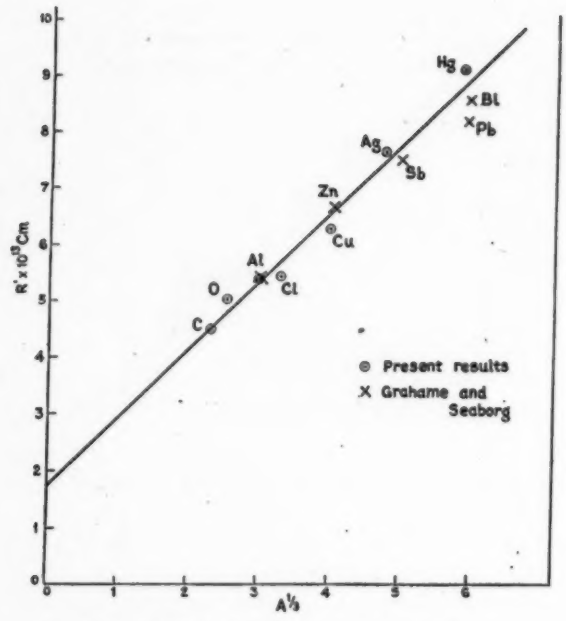

FIG. 4. Measured values of nuclear radius *vs.* (atomic mass number)$^{\frac{1}{3}}$.

for neutrons elastically scattered into the detector by the absorber. (Weisskopf's formula[9] indicates that a negligibly small number of inelastically scattered neutrons have energies greater than 20 Mev.) The transmission equation under these conditions may be expressed by

$$I = I_0 a(x) \exp\left[-N(\sigma_i+\sigma_e)x\right] \times\{1+Ax+Bx^2+\cdots\}, \quad (5)$$

where $a(x)$ represents the correction for obliquity of neutron paths, and A and B are constants giving the contribution to the measured intensity of singly and doubly scattered neutrons, respectively. A and B are determined by the differential scattering cross section $\sigma(\varphi)$, given by Eq. (2), by the sensitivity of the detector for the scattered neutrons, and by the geometry. Formulae for making these corrections were developed by Dr. C. L. Critchfield and gave the results described below.

The obliquity factor $a(x)$ has a value of 0.998 for the thickest scatterer and can therefore be replaced by unity. The single scattering coefficient A is about 17 percent of $N(\sigma_i+\sigma_e)$ for carbon and increases to a value of 40 percent for Hg. These corrections are large because the diffraction formula gives an elastic scattering predominantly in the forward direction; if the scattering were spherically symmetrical, A would not exceed 3 percent. The double scattering coefficient B is negligible except for Ag and Hg; for Hg, the contribution of doubly scattered neutrons is 38 percent of the single scattering. The total cross sections calculated from Eq. (5) are given in the last column of Table I. The calculations include the assumption that $\sigma_e=\sigma_i=\pi R'^2$.

DISCUSSION

(a) Hydrogen

Dr. C. L. Critchfield calculated the neutron-proton cross section for 25 Mev neutrons for the case of *s*-scattering[1] and obtained a value of 0.345×10^{-24} cm^2 for the cross section to be observed under the present geometrical conditions. Similar calculations based on Rarita and Schwinger's symmetrical, charged, and neutral meson theories[16] gave values of 0.395, 0.43, and 0.89×10^{-24} cm^2, respectively. These calculations include the effect of *p*-wave scattering as well as the *s*-wave interaction. The theoretical cross sections are to be compared with the observed value of $0.39\pm0.03\times10^{-24}$ cm^2. The agreement with the symmetrical meson theory is quite good. However, the experimental accuracy is not adequate to exclude the *s*-wave and charged meson theories. The neutral meson theory gives a cross section which is much too large.

(b) Other Nucleii

As noted above, the cross sections for nuclei other than the proton are the sum of the elastic

TABLE II. Comparison of the nuclear radii derived from the present experiment with the results of Grahame and Seaborg.

Element	A	$A^{\frac{1}{3}}$	$R'\times10^{13}$ cm Present values	$R'\times10^{13}$ cm Grahame & Seaborg
C	12.0	2.29	4.54	
O	16.0	2.52	5.05	
Al	27.0	3.00	5.43	5.45
Cl	35.5	3.28	5.47	
Cu	63.6	3.99	6.31	
Zn	65.4	4.03		6.70
Ag	107.9	4.76	7.69	
Sb	121.8	4.96		7.55
Hg	200.6	5.86	9.15	
Pb	207.2	5.92		8.24
Bi	209.0	5.94		8.61

[16] W. Rarita and J. Schwinger, Phys. Rev. **59**, 556 (1941).

plus inelastic scattering cross sections. Since each is taken to be equal to $\pi R'^2$, the collision radius is given by $R'=(\sigma/2\pi)^{\frac{1}{2}}$. The values of R' obtained from the last column of Table I are tabulated in the fourth column of Table II. The first, second, and third columns give the element, the corresponding atomic mass number, and the cube root of the latter.

The last column in Table II gives the values of R' obtained from the "absorption plus inelastic scattering" cross sections found by Grahame and Seaborg[8] for neutrons with energies greater than 7 Mev. Since their method eliminates the detection of elastic scattering, their cross sections are equal to $\pi R'^2$. Both sets of data are presented graphically in Fig. 4, giving R' as a function of $A^{\frac{1}{3}}$. The agreement between the two sets of values is within the observational accuracy of 5 percent. It is, however, difficult to estimate the accuracy of the absolute values of R for the heavier elements, since the geometrical corrections were quite large.

It is evident from Fig. 4 that a straight line through the experimental points has a positive intercept, as suggested by Eq. (3), with $b=1.7\pm0.4\times10^{-13}$ cm and $r_0=1.22\pm0.15\times10^{-13}$ cm.

Frankel's analysis[11] of elastic scattering data[12] yielded the values $b=2.3\times10^{-13}$ cm and $r_0=1.25\times10^{-13}$ cm, in good agreement with our results. However, other methods of determining r_0 give larger values. Weisskopf and Ewing[3] derived a value of 1.3×10^{-13} cm from a study of p-n reactions in the neighborhood of Cu, while the Coulomb energy differences of light nuclei gave 1.35×10^{-13} cm. Better agreement is found with r_0 calculated by Present[5] from known mass defects, 1.2–1.3×10^{-13} cm. The disagreement between various methods is undoubtedly due in part to uncertainties in the interpretation of the interactions being studied. Present[5] has suggested that the light and heavy nuclei are less dense than those in the region $A=50$. The present data seem to indicate such a trend, but the measurements are not sufficiently accurate to warrant any definite conclusions on this point.

The agreement between the cross sections measured at 25 Mev, and at 14 Mev by Grahame and Seaborg, indicates that the density of levels reached in the excitation of nuclei with $A\geqslant27$ by neutrons of 14 Mev is sufficiently high to prevent the appearance of important resonances. The carbon cross sections measured by Salant and Ramsey for 14 and 15 Mev neutrons are 1.27 ± 0.04 and $1.36\pm0.05\times10^{-24}$ cm^2, in close agreement with our value of 1.29×10^{-24} cm^2, at 25 Mev, indicating that the above-mentioned condition may be satisfied for the very light nuclei at 14 Mev.

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X-Ray Line Absorption in the *M*-Series for 62-Sm*

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The M_{IV} and M_V absorption edges for 62-Sm were studied by use of evaporated films of Sm_2O_3 and $SmCl_3$. In view of data in the literature on *M*-absorption of the rare earths 70-Yb and 68-Er, it seemed possible that absorption lines would be found corresponding to transitions in which electrons from the *M*-shell are lifted to the inner N_{VII} and N_{VI} levels. When thin absorption films were employed, the entire absorption had the appearance of lines, but thick films revealed the definite edge-like character of the absorption. The M_{IV} and M_V edges are very similar and are both complex, four edges being distinguished in each case. In both cases the first pair of edges is separated from the latter pair by a minimum, which may correspond to transitions in which the electrons are lifted to the N_{VII} and N_{VI} levels.

INTRODUCTION

THE possibility of x-ray "line absorption" corresponding to the transitions $M_{IV}N_{VI}$ and M_VN_{VII} enhances the interest of a study of the M_{IV}- and M_V-absorption edges for the elements of the rare earth group.[1,2] The present work reports measurements for the M_{IV}- and M_V-edges for 62-Sm.

EXPERIMENTAL

Apparatus

A curved mica crystal having a short radius of curvature, 498 mm, and a large area for reflection, 1.5×20 mm, was employed in the spectrograph in order to shorten the time required for exposure in the study of absorption edges at short wave-lengths. A cathode-ray oven, an integral part of the spectrograph, served for preparation of absorption films by evaporation of the substances to be studied and condensation on aluminum foil, 0.6μ thick. Details of construction of the apparatus have been previously described.[3]

Operation

When exposing for the absorption edges the effective voltage was maintained at 1.6 kv, which is less than two times the excitation potential for the M_V-edge. Because the high-potential circuit was not provided with condensers, some second-order radiation was excited at the potential peaks, but third-order radiation was not excited. A tungsten anticathode provided the continuous radiation. The reference substances were KCl ($19-K\ K\alpha_1\lambda=3733.68$ x.u.) and ZnO ($30-Zn\ K\alpha_1\lambda=1432.17$ x.u.).[4]

Sm_2O_3 was prepared from samarium nitrate. $SmCl_3$ was prepared from the oxide by heating at 600°C in a current of S_2Cl_2. Films of Sm_2O_3 were made by evaporating the oxide from a crucible of Mo or graphite. Films of $SmCl_3$ were formed by evaporation from a crucible of Mo. The $SmCl_3$ was not entirely free from oxide. Inasmuch as evaporation of the chloride required only a power output of about 10 watts, while evaporation of the oxide required at least 200 watts, contamination of the film of $SmCl_3$ with evaporated oxide was not probable. The thickness of each film was determined after the absorption measurements had been carried out by weighing the film and its aluminum base.

Since it would have been of interest to observe the absorption edges for metallic Sm, attempts were made to prepare a film of the metal by reduction of the oxide in the crucible and subsequent evaporation of the metal. Because of its high volatility, reduction of $SmCl_3$ in the high vacuum required for the operation of the cathode-ray oven was not attempted. During the short

* This work was aided by the G. Hilmer Lundbeck fellowship of the American Scandinavian Foundation.

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[1] Ernst Lindberg, Dissertation, Nova Acta Reg. Soc. Sci. Uppsalenius 7, Ser. IV, No. 7 (1931).

[2] Sandström, Dissertation, Nova Acta Reg. Soc. Sci. Uppsalenius 9, Ser. IV, No. 2 (1935).

[3] Kenneth C. Rule, Phys. Rev. 66, 199 (1944).

[4] Manne Siegbahn, *Spektroskopie der Röntgenstrahlen* (Verlagsbuchhandlung Julius Springer, Berlin, 1931), second edition.

time available for pursuing these experiments satisfactory results were not attained with the oxide.

All measurements were made on photometer curves with an abscissa enlargement of 3×. A millimeter netting was copied onto the paper and the enlargement was determined by making a photometer curve of scratches on a plate, whose distances apart had been measured in a comparator. The photometer curves could be read to 0.1 mm = 0.03 mm on the plate = 1.1 x.u. (34 x.u./mm). Inasmuch as the full opening (18–20 mm) of the crystal was employed at the glancing angle of 36°, a focusing defect of 4.7 x.u. was to be expected,[2] but observation of the shifts of the reference lines when 20-mm and 8-mm openings and an abscissa enlargement of 10× were employed showed that the wave-length error introduced by the focusing defect did not exceed 0.4 x.u.

Data and Discussion

The first plates exposed, employing films of SmO_2, bore the appearances of "line absorption." One strong absorption line, preceded by a weaker line at longer wave-lengths, was found for each of the edges, M_V and M_{IV}. Because plates exposed from subsequent films had a different appearance, however, the "lines" having become broader, it was suspected that the results depended upon the thickness of the absorption film. Since the thickness could not be measured accurately, a series of four plates was exposed from an absorption film whose thickness was increased between each exposure by further evaporation of Sm_2O_3. The evaporation times were chosen so as to increase the thickness approximately in the ratios 1:3:10:30. The final thickness of the film was found to be 0.8 mg/cm². The photometer curves made from these four plates are reproduced in Fig. 1.

A complete gradation from an appearance of "line absorption" to true edges is apparent in this series. Further, the absorption for the weak "lines" on the long wave side of the main ones increases relatively as the thickness increases, whereby one or two long wave components of the edge arise. It is obvious that the absorption cannot depend in principle upon the film thickness, but the absorption for each different wave-length is simply more or less dominant depending on the relations between the absorption coefficients and the film thickness.

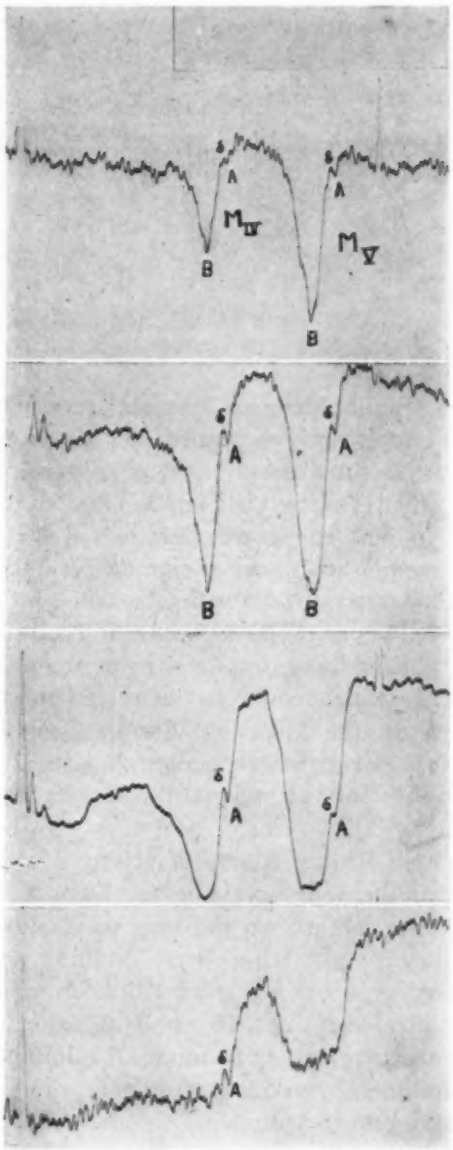

FIG. 1. Photometer curves of M_{IV} and M_V absorption edges of Sm in Sm_2O_3. Thickness of absorption film increases successively from uppermost to lowermost curve.

An entirely analogous group of plates, with absorption varying from line-like to edge-like, was obtained also for $SmCl_3$ absorption films of varying thicknesses. In the case of this substance a new absorption film was produced for each new plate, in order to avoid the danger that a single film, whose thickness was periodically aug-

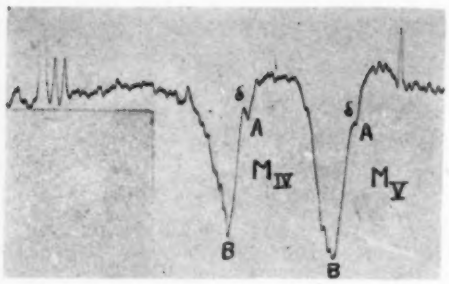

FIG. 2. Photometer curve of M_{IV} and M_V absorption edges of Sm in $SmCl_3$, thin absorption film.

mented, might slowly pick up moisture under a series of long runs. A photometer curve corresponding to the absorption by a relatively thin film of $SmCl_3$ is shown in Fig. 2. The wave-length values for the edges are recorded in Table I. There are no differences of significance between the values obtained with Sm_2O_3 and $SmCl_3$ as absorption films. The agreement is best, in particular, for those details, such as the minima *A* and *B*, which could be measured most accurately. In the following discussion only the data for the oxide will be quoted, but the discussion applies just as well to the results for the chloride.

For each of the edges, M_{IV} and M_V, there appear to be four components. Two of these precede the minima on the long wave side and two follow them. With a spectrograph of greater dispersion or a photographic emulsion of grain size smaller than Agfa 18°/10 Din, other components might possibly be found. A minimum on the photometer curves corresponds, of course, to a maximum in absorption.

The similarity between the M_{IV}- and M_V-edges is striking. For the M_V-edge the minima *A* and *B* differ by 51 x.u., for the M_{IV}-edge by 47 x.u. The first edge, K_1, precedes the minimum *B* by 71 x.u. for the M_V-edge, and by 64 x.u. for the M_{IV}-edge. This similarity indicates that the end levels for M_{IV}- and M_V-absorption are identical.

These results can be compared with Lindberg's data[1] for the $M_{IV}N_{VI}$- and M_VN_{VII}-emission lines for Sm (Table I). The components of the β-line correspond to the $M_{IV}N_{VI}$-transition, those of the α-line to the M_VN_{VII}- and perhaps M_VN_{VI}-transitions. For both the M_{IV}- and M_V-edges absorption begins at a wave-length corresponding to the longest wave components of the β- and α-lines. It is remarkable that the minimum, *A*, in the M_{IV}-edge has almost exactly the same wave-length as the β_I (or $M_{IV}N_{VI}$) line, which may indicate that this minimum is an "absorption line." However, none of the emission lines that have been observed in the α-group corresponds to the minimum, *A*, in the M_V-edge. Table II gives the average wave-length values and the v/R-values for the edges as well as the energy values for the M_{IV}- and M_V-levels as calculated by Raoult[5] from her measurements of the L_{III}-edge for Sm and Siegbahn's values for the *L*-emission lines.[4] The comparison shows, as usual, that the measured values of the energy levels are lower (about 1 percent) than the calculated values.

Similar measurements would have to be made for other rare-earth elements, especially for 63-Eu, for which the β_{II} and β_{III} lines are missing, to determine definitely whether or not the correspondence between the minima *A* and the long wave satellite β_I represents an absorption transition in which the electrons are lifted to the N_{VI} level. This investigation has shown that with homogeneous absorption films of the proper thicknesses the M_V- and M_{IV}-edges for an element of as low an atomic weight as 62-Sm can

TABLE I. Wave-length values in *x* units for the M_{IV}- and M_V-edges and *M*-emission lines for 68-Sm. Figures in parentheses represent the number of measurements (plates) included in the average; the preceding figures show the average deviations from the averages.

	Sm_2O_3			$SmCl_3$			Sulfate	
				M_V-edge			M_V-emission lines[1]	
K_1	11531.1	1.0	(4)	11534.8	5.4	(2)	α_{III}	11532
K_2	11519.4	1.8	(7)	11516.8	1.2	(3)		
A	11511.1	2.7	(8)	11510.2	1.4	(3)		
δ	11500.5	1.6	(7)	11504.7	0.7	(3)		
K_3	11493.7	1.8	(7)	11494.7	4.1	(4)		
K_4	11482.1	3.6	(7)	11482.0	6.3	(4)	α_{II}	11475
B	11460.1	1.4	(4)	11459.8	0.5	(2)	α_I	11406
				M_{IV}-edge			M_{IV}-emission lines[1]	
K_1	11267.6	2.3	(6)	11270.1	2.9	(3)	β_{III}	11271
K_2	11255.9	1.9	(6)	11258.8	2.1	(3)	β_{II}	11254
A	11250.4	0.8	(7)	11250.4	0.7	(4)		
δ	11240.1	2.1	(7)	11241.3	0.7	(3)	β_I	11238
K_3	11232.3	3.1	(7)	11228.0	3.1	(3)		
K_4	11224.4	1.5	(4)	11217.8		(1)		
B	11203.2	1.4	(6)	11204.9	0.0	(2)		
							β'	11147

[5] Mme. Raoult, Comptes rendus **207**, 434 (1939).

be brought out with great contrast. Furthermore, the observed dependence of the appearance of the absorption edges upon the thickness of the absorption film may bring into question whether the "absorption lines" found by Lindberg for the M_V-edges for 70-Yb and 68-Er were actually due to line-absorption or to the use of excessively thin absorption films.

TABLE II. Average wave-lengths and ν/R-values for the edges and calculated energy levels for 68-Sm.

	M_V			M_{IV}	
	x.u.	ν/R		x.u.	ν/R
K_1	11532.3	79.02	K_1	11268.4	80.87
K_2	11518.6	79.11	K_2	11256.9	80.95
A	11510.8	79.17	A	11250.4	81.00
δ	11501.8	79.23	δ	11240.5	81.07
K_3	11491.1	79.28	K_3	11231.1	81.14
K_4	11482.1	79.36	K	11223.1	81.20
B	11660.0	79.52	B	11203.6	81.34
M_V-level (Raoult[5])		79.94	M_{IV}-level[5]		82.12

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A New Approach to Kinematic Cosmology

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The kinematical aspect of relativistic cosmology is examined on the basis of three postulated requirements: The constancy of the velocity of light, spatial isotropy, and homogeneity. Three distinct types of cosmological models are obtained, characterized by different motions of nebulae. The metric of any universe is conformal to Minkowski space and Maxwell's equations are the same for all possible universes. In Part II, it is shown that the cosmological models are metrically, though not topologically, equivalent to those of H. P. Robertson. Next, special models are examined and their line elements brought into the conformal-Minkowskian form. The problem of the displacement of the lines of nebular spectra is discussed; formulas are obtained and applied to some special cosmological models. Finally, idealized experiments are described which indicate the physical content of the cosmological coordinates.

I. General Theory

INTRODUCTION

BY kinematical cosmology is usually understood that part of relativistic cosmology which deals with the metric form of our universe, characterized by a four-dimensional space-time manifold, and with the motion of free particles and light rays in this universe. In this sense the present paper deals with kinematical cosmology and ignores its dynamical aspect, i.e., the connection between the Riemannian curvature tensor on the one hand and the energy-momentum tensor on the other.

The possible mathematical models of the universe are usually deduced from a few simple and convincing principles. In such models the nebulae are represented by freely moving particles of a fluid and we imagine such a particle at every point in space. Each of these particles moves along a geodesic line. Such a particle, representing a nebula, will be called a *fundamental particle* and an observer moving with it a *fundamental observer*.[1] The most important concept by which cosmological models will be described and characterized is that of fundamental particles and their motion. Next, light rays or geodesic null lines must be considered; they are the messengers of world events between fundamental particles. Finally, the universe is characterized by a Riemannian metric. The metric form and the motion of fundamental particles fully determine the kinematical behavior of the universe. It might seem that the motion of fundamental particles can be deduced immediately from the Riemannian metric as the particles move along geodesics. But this is not so. A geodesic line is, at each world-point, determined by its direction there, whereas the world-line of a fundamental particle is completely determined by the choice of a world-point. The world-line of a fundamental particle is a geodesic; but the converse statement is not necessarily true.

The questions which this paper tries to answer are: What are the possible metric forms describing a universe? What are the possible motions of fundamental particles? The guiding principle, commonly accepted and leading to a solution of these problems, is the *principle of homogeneity*, sometimes called the uniformity or the cosmological principle. It states that every fundamental observer sees and describes the world in the same way.

Traditionally, the problem of relativistic cosmology[2] is attacked by choosing a coordinate system in which all fundamental particles are at rest. The fact that such a coordinate system exists is by no means obvious and is closely

[1] These terms seem to be originally due to E. A. Milne, *Relativity, Gravitation, and World-Structure* (Clarendon Press, Oxford, 1935).

[2] An excellent account of "Relativistic Cosmology," including a bibliography complete up to 1932, is to be found in H. P. Robertson's article, Rev. Mod. Phys. 5, 62–90 (1933); also R. C. Tolman, *Relativity Thermodynamics and Cosmology* (Clarendon Press, Oxford, 1934).

related to the principle of homogeneity. Thus the line elements discussed in cosmology are usually of the form

$$ds^2=d\tau^2-R^2(\tau)d\sigma^2. \tag{0.1}$$

Here $R(\tau)$ is an arbitrary function of time and $d\sigma^2$ is the metric of a three-dimensional space of constant curvature $k=1$, -1, or 0. Thus the universe is characterized by an arbitrary function $R(\tau)$ and by the choice of one among three possible spaces. The problem of the motion of fundamental particles disappears from such a presentation because these particles are always at rest. It is the *space structure*, i.e. the curvature $kR^{-2}(\tau)$ of the 3-space $\tau=$constant, which characterizes the cosmological model. It should be noted that, in the coordinate system of (0.1), the speed of light in any fixed direction is a function of time and depends on both k and $R(\tau)$.

Obviously, a discussion in which the four-dimensional universe is characterized chiefly by the curvature of a three-dimensional space is contrary to the spirit of relativity theory in which the world is represented by a four-dimensional space-time continuum. Historically, this approach goes back to Einstein's first cosmological paper,[3] to the Einstein universe of which all others seem to be natural generalizations. This point of view, based on dynamic considerations and on the generalization of the gravitational equations, was long ago abandoned by its originator.

We believe that a deeper insight into cosmological problems is gained by a new approach. Relativistic cosmology, at least in its kinematical aspect, should form a link between the restricted and general theories of relativity. In restricted relativity the world is represented by a Minkowski continuum. This is one of many cosmological backgrounds which satisfy the principle of homogeneity. We shall see that an approach to cosmology is possible in which the structure of a three-dimensional space does not enter the picture. We believe that this new approach puts into the foreground the more essential concepts of kinematical cosmology, i.e., the type of motion of fundamental particles, rather than the space structure.

The ideas which have been sketched above will become clearer if we now summarize some of our results.[4]

Every cosmological background is a Riemannian manifold with a metric of the form

$$ds^2=\gamma(t,r)(dt^2-dx^2-dy^2-dz^2),\qquad r^2=x^2+y^2+z^2. \tag{0.2}$$

We shall see that γ is not an arbitrary function of t and r; but for the conclusions which we shall now draw the particular form of γ does not enter the argument. A coordinate system in which the Riemannian metric has the form (0.2) will be called a *cosmological coordinate system*, or, briefly, a c.c.s.

Thus every cosmological background is conformal to a Minkowski background. Physically, the light geometry is that of a flat Minkowski continuum. The line element (0.2) appears as a natural generalization of the Minkowski space in restricted relativity.

We may, however, interpret (0.2) in a different way and say that every cosmological background differs from a Minkowski background only by a gauging function determining the behavior of clocks and measuring rods. This statement requires some amplification. Starting from (0.2), and without transforming the coordinate system, we introduce new clocks and measuring rods by means of the gauge transformation,[5]

$$ds'=\lambda ds;\quad \lambda=\gamma^{-\frac{1}{2}}.$$

We then have $\gamma'=\gamma^{-1}\gamma=1$, and the vector field, characterizing the gauging (i.e., the metrical connection), changes from $\chi_i=0$ to

$$\chi_i'=\partial(\log\lambda)/\partial x^i=-\tfrac{1}{2}\partial(\log\gamma)/\partial x^i.$$

We can, therefore, characterize a cosmological space by

$$ds'^2=dt^2-dx^2-dy^2-dz^2,\qquad \chi_i'=-\tfrac{1}{2}\partial(\log\gamma)/\partial x^i, \tag{0.3}$$

i.e., by a Minkowski metric and an integrable

[3] A. Einstein, "Kosmologische Betrachtungen zur Allgemeinen Relativitätstheorie," Sitz. Preuss. Akad. Wiss. 142–152 (1917).

[4] L. Infeld, "A New Approach to Relativistic Cosmology," Nature 156, 114 (1945).

[5] H. Weyl, *Space, Time, Matter* (Methuen & Company, Ltd., London, 1922), Chapter II, Section 16.

gauging field χ_i'. This is a new geometric picture and, though (0.3) can always be replaced by (0.2), this new interpretation is very suggestive as it abandons all discussion of curved, expanding universes and shifts the responsibility for cosmological phenomena to the gauging field, the gradient of a scalar field, which determines the behavior of clocks and measuring rods.

It should be added that this interpretation, though based on Weyl's famous work, has nothing in common with Weyl's unified field theory. No connection is assumed between the vector χ_i and the vector potential of an electromagnetic field.

The next conclusion which may be drawn from (0.2) and (0.3) is that Maxwell's equations are the same for flat space as for any cosmological space.[6] This immediately follows from (0.3) and from the fact that Maxwell's equations are gauge-invariant. But for the sake of clarity this simple conclusion will be deduced from (0.2). We write Maxwell's equations for empty space in the usual form

$$\begin{gathered}\partial F_{ij}/\partial x^k+\partial F_{jk}/\partial x^i+\partial F_{ki}/\partial x^j=0,\\ \partial((-g)^{\frac{1}{2}}F^{ij})/\partial x^j=0.\end{gathered}\tag{0.4}$$

We have

$$\begin{aligned}(-g)^{\frac{1}{2}}F^{ij}&=(-g)^{\frac{1}{2}}g^{ik}g^{jl}F_{kl}=\gamma^2(1/\gamma^2)\eta^{ik}\eta^{jl}F_{kl}\\ &=\eta^{ik}\eta^{jl}F_{kl},\end{aligned}\tag{0.5}$$

where

$$\eta_{ij}=\eta^{ij}=\begin{pmatrix}1&0&0&0\\0&-1&0&0\\0&0&-1&0\\0&0&0&-1\end{pmatrix}.$$

Therefore Maxwell's equations, which are differential equations for F_{ij}, have the following form both for a Minkowski and a cosmological background:

$$\begin{gathered}\partial F_{ij}/\partial x^k+\partial F_{jk}/\partial x^i+\partial F_{ki}/\partial x^j=0,\\ \eta^{ik}\eta^{jl}\partial F_{kl}/\partial x^j=0.\end{gathered}\tag{0.6}$$

Another problem which suggests itself here is that of Dirac's relativistic equations for an electron. Indeed, much work was done in solving Dirac's equations for cosmological spaces. From our point of view, the problem, properly formulated, is the following: Are Dirac's equations, like Maxwell's equations, insensitive to the choice of the γ function or not? The answer to this question is not as simple as in the case of Maxwell's equations and requires a special investigation which will be given elsewhere.

The most obvious conclusions resulting from the fact that the cosmological background can be represented by (0.2) or (0.3) have been formulated above. It has been seen how the problem of space structure disappears if the c.c.s. is used and it will be shown how the problem of the motion of fundamental particles appears instead. In a c.c.s. the fundamental particles are, in general, not at rest. Three types of motion are possible, namely, the oscillating motion, the converging-diverging motion, and the simple case of rest. To each permissible form of the function γ there belongs at least (and, in general, exactly) one kind of motion. This motion, and not the space structure, characterizes the universe. It is the study of the permissible functions γ and the associated motions which forms the chief content of this paper.

1. DERIVATION OF THE LINE-ELEMENTS

The derivation of the metrical forms which describe the universe as a whole will be based on three distinct postulates. All three assumptions have immediate physical significance. The last two are well justified by recent astronomical observations. It is not claimed that the first postulate is completely independent of the other two; however, as it has a simple physical content, we prefer to introduce it as a separate requirement.

Postulate I *on light-geometry.*—There exists a coordinate system such that the geometry of light rays is the same as in flat Minkowski space; i.e., light rays travel along straight lines with constant velocity $c=1$, say.

Mathematically, this implies that the four-space must be conformal to Minkowski space, i.e., the line element may be written in the form

$$ds^2=\gamma(dt^2-dx^2-dy^2-dz^2),\tag{1.1}$$

$$=\gamma\eta_{ij}dx^idx^j,\tag{1.11}$$

[6] This conclusion seems to be unknown. Compare E. Schrödinger, "Maxwell's and Dirac's Equations in the Expanding Universe," Proc. Roy. Irish Acad. **46A**, 25–47 (1940).

where γ is a function of t, x, y, z. In (1.11) the usual summation convention applies to double indices,[7] x^i $(i=0, 1, 2, 3)\equiv(t, x, y, z)$.

Postulate II *on isotropy.*—The universe, characterized by the line element (1.1), is spatially isotropic.

Mathematically, the cosmological line element must be invariant in form under a 3-parameter continuous group of rotations which leaves every point on the t axis fixed. An immediate consequence is that γ is a function of t and $x^2+y^2+z^2$ only:

$$\gamma=\gamma(t, r), \quad r^2=x^2+y^2+z^2. \tag{1.2}$$

The assumption of isotropy is well justified by nebular counts,[8] which indicate that the distribution of nebulae, as observed from the earth, exhibits spherical symmetry when averaged on a sufficiently wide scale.

If suitable assumptions are made as to the absolute average magnitude and the luminosity of nebulae, and the density distribution of internebular matter, as yet unobservable, recent astronomical research[8] indicates that, on a scale large compared to the mean distance between nebular clusters, matter is uniformly distributed throughout the universe. Thus, neglecting local irregularities, the homogeneity of the material universe leads, by what is usually known as Mach's principle, to the following:

Postulate III *on homogeneity.*—The view of the universe is the same for every fundamental observer.

In our model of the universe we have fundamental particles, each moving along a geodesic line. Our original coordinate system is related to one fundamental particle; by this is meant that the geodesic world-line of this particle is the t axis $x=y=z=0$. If we now change the coordinate system, relating it to any other fundamental particle, our view of the universe remains unchanged; in particular, ds^2 is invariant in form.

Mathematically, the line element ds^2 must be invariant in form under a continuous group of transformations, moving the t axis into world-lines different from it. The existence of such a group of coordinate transformations restricts the possible functions $\gamma(t, r)$. In the following we find line elements which satisfy our homogeneity postulate. However, the treatment adopted here gives no indication as to whether the admissible forms have been exhausted or not. In the appendix, the powerful methods of the theory of continuous groups are applied to the problem and it is there shown that the simpler arguments of this section do, indeed, yield essentially all possible line elements.

Consider the hyperquadric

$$z_0^2-z_1^2-z_2^2-z_3^2-(1/K)z_4^2=-(1/K), \quad K=\text{constant} \tag{1.3}$$

in five-space and project stereographically from the pole $(0, 0, 0, 0, -1)$ onto the tangential hyperplane $z_4=1$. If the point $(z_0, z_1, z_2, z_3, z_4)$ on the hyperquadric is projected into $(x^0, x^1, x^2, x^3, 1)$, a simple calculation yields the equations

$$z_i=\frac{x^i}{1-Ka/4}, \quad z_4=\frac{1+Ka/4}{1-Ka/4}, \quad i=0, 1, 2, 3, \tag{1.31}$$

where

$$a=\eta_{ij}x^ix^j=t^2-r^2. \tag{1.311}$$

It is well known that, for a suitable metric, the stereographic projection is conformal. Thus we obtain, by differentiating (1.31), the following identity:

$$f(z_0, z_4)(dz_0^2-dz_1^2-dz_2^2-dz_3^2-(1/K)dz_4^2)= \gamma(t, r)(dt^2-dx^2-dy^2-dz^2)=ds^2, \tag{1.32}$$

where

$$\gamma(t, r)=f(z_0, z_4)/(1-Ka/4)^2, \tag{1.33}$$

z_0, z_4 being expressed in terms of t, r by (1.31). The problem of finding transformations leaving (1.1) invariant is obviously equivalent to that of finding transformations of the five-dimensional z space which leave (1.3) and the left-hand side of (1.32) invariant in form.

One such transformation group, if f is a function of z_0 only, is that of linear transformations of the four variables z_1, z_2, z_3, z_4 leaving $z_1^2+z_2^2+z_3^2+(1/K)z_4^2$ invariant, with $z_0\to z_0$. The group is, for positive K, that of real orthogonal transformations, i.e., Euclidean rotations, of the

[7] Latin indices will consistently range over 0, 1, 2, 3, while Greek indices will range over 1, 2, 3.

[8] A comprehensive account of astronomical observations of cosmological interest is to be found in E. P. Hubble, *The Observational Approach to Cosmology* (Clarendon Press, Oxford, 1937).

variables $z_1, z_2, z_3, K^{-\frac{1}{2}}z_4$; for negative K, that of Lorentz transformations of the "time-like" variable $(-K)^{-\frac{1}{2}}z_4$ and the "space-like" variables z_1, z_2, z_3. Thus $f=f(z_0)$ yields, by (1.33), the line elements

$$ds^2=(1-Ka/4)^{-2}f(t/(1-Ka/4))\eta_{ij}dx^i dx^j, \quad (1.4)$$

which satisfy our three postulates. The complicated transformations (see appendix) leaving this form invariant appear, by the above, simply as "rotations" of a four-dimensional subspace about a line (the z_0 axis) in a five-dimensional manifold.

The metric form (1.4) is admissible for all non-zero values of K. It is natural to consider the limiting case when $K=0$, and

$$ds^2=f(t)\eta_{ij}dx^i dx^j. \quad (1.41)$$

The limiting process $K\to 0$ is difficult to carry out on the five-dimensional linear transformations of the z considered above. However, it is immediately obvious that the line element (1.41) conforms to the homogeneity requirement as it is invariant under spatial translations

$$(t, x, y, z)\to(t, x+\xi, y+\eta, z+\zeta). \quad (1.42)$$

We shall formally include (1.41) in the metric forms (1.4) by permitting K to become zero. Then, as will be shown in the following and in the appendix, the metric forms (1.4) describe all possible universes which satisfy our three postulates. The line elements (1.4) can be written in the form

$$ds^2=f(t/(1-Ka/4))ds_0^2, \quad (1.43)$$

where ds_0^2 is the line element of an indefinite 4-space of constant curvature $-K$. This may be contrasted with the forms (0.1) where a 3-space of constant curvature is multiplied by an arbitrary function (of time).

We shall now obtain two further metric forms which are admissible. However, as they are obtained from some of the line elements (1.4), for $K\leq 0$, by coordinate transformations, they do not yield new universes and are essentially equivalent to those included above.

If $K<0$, then z_0^2 and $(-1/K)z_4^2$ have the same signature in (1.3) and in (1.32). Thus we may interchange those two variables. Then f is a function of z_4 only, i.e., of a; (1.3) and the left-hand side of (1.32) are invariant under Lorentz transformations of the variables z_0, z_1, z_2, z_3 with $z_4\to z_4$, i.e., with $a\to a$. By (1.31), we see that such transformations of the five-space are simply the Lorentz transformations in our original space of variables x^i. The new line elements are, by (1.33), of the form

$$ds^2=\gamma(a)\eta_{ij}dx^i dx^j, \quad (1.5)$$

where

$$\gamma(a)=(1-Ka/4)^{-2}\times f\left(-(-K)^{-\frac{1}{2}}\frac{1+Ka/4}{1-Ka/4}\right). \quad (1.51)$$

From the way in which it was obtained, it is clear that (1.5) is equivalent to (1.4; $K<0$). However, (1.5) is much simpler than the previous forms (1.4), and this is also true of the group of coordinate transformations which leave the line element invariant and hence of the motion of fundamental particles. Thus we shall, whenever possible, prefer to characterize universes by the metric (1.5) rather than by (1.4) if $K<0$.

The last cosmological line element which we derive is equivalent to (1.41), where $K=0$. It is more cumbersome than (1.41) and need not be discussed in detail. However, we include it in this section for the sake of completeness. The inversion

$$(t, x^\sigma)\to(-t/a, x^\sigma/a), \quad \sigma=1, 2, 3, \quad (1.61)$$

is a conformal transformation of Minkowski space and changes (1.41) into the form[9]

$$ds^2=(1/a^2)f(-t/a)\eta_{ij}dx^i dx^j. \quad (1.6)$$

The minus-sign in the inversion (1.61) is necessary to preserve the sense of time. The group of transformations which leave (1.6) invariant are obtained from the spatial translations (1.42) by subjecting them to the inversion (1.61).

This completes our survey of the cosmological line elements satisfying our three requirements. The forms (1.4), (1.5), (1.6), are shown, in the appendix, to exhaust all possibilities, except for a trivial change of the temporal origin, i.e., except

[9] It may be noted that if (1.4) be written in the form $ds^2=\phi(Kt/4(1-Ka/4))(K/4)^2(1-Ka/4)^{-2}\eta_{ij}dx^i dx^j$, then, if the function ϕ does not involve the parameter K implicitly, we obtain (1.6) from (1.4) by the limiting process $K\to\infty$.

TABLE I. The cosmological line elements.

Case	K ($\alpha>0$)	γ	Transformations preserving ds
I	$1/\alpha^2$	$(1-a/4\alpha^2)^{-2}f(t/(1-a/4\alpha^2))$	Rotations in $(z_1, \alpha z_4)$ plane
II	$-1/\alpha^2$	$(1+a/4\alpha^2)^{-2}f(t/(1+a/4\alpha^2))$	Lorentz transf. in $(z_1, \alpha z_4)$ plane
II′		$\gamma(a)$	Lorentz transformations of x^i
III	0	$f(t)$	Spatial translations of x^i
III′		$(1/a^2)f(-t/a)$	From III by inversion

for the forms obtained from them by the transformation $t\to t+$constant.

We summarize the results of this section in Table I.

2. THE MOTION OF FUNDAMENTAL PARTICLES

The postulate of homogeneity ensures the existence of coordinate changes which leave the metric form invariant and transform the t axis into other world-lines which are the t axes of the new coordinate systems. It follows from the isotropy of the line element, which must hold in all equivalent coordinate systems, that these world-lines are geodesics. Thus it is natural to identify such geodesics with the world-lines of the fundamental particles. If $x^i\to x'^i(x)$ is a transformation leaving (1.1) invariant, then the equations of motion of the fundamental particle, which is related to the new coordinate system, are

$$x'^\sigma(t, x, y, z)=0, \quad \sigma=1, 2, 3. \tag{2.1}$$

The spatial isotropy of the universe implies that the world-line of a fundamental particle must, in cosmological coordinates, lie in a plane through the t axis; i.e., the fundamental particles move radially. Thus we may, without loss of generality, consider fundamental world-lines in the tx plane only, and, throughout this section, assume the two equations of motion

$$y=z=0. \tag{2.2}$$

The third equation, describing the motion along the x axis, will be obtained and discussed in each of the five cases enumerated in Table I.

Case I—$K=1/\alpha^2>0$.—We need consider only Euclidean rotations in the $(z_1, \alpha z_4)$ plane, since, by (1.31), $y=z=0$ implies $z_2=z_3=0$. Thus, suppressing two dimensions, the rotations about the z_0 axis and the paths of fundamental particles can easily be visualized by means of a diagram.

The t axis $x=0$ has, in the z space, the parametric equations

$$z_0=t/(1-t^2/4\alpha^2), \quad z_1=0,$$
$$z_4=(1+t^2/4\alpha^2)/(1-t^2/4\alpha^2), \tag{2.3}$$

by (1.31). The equations are those of a rectangular hyperbola in the $(z_0, \alpha z_4)$ plane, as shown in Fig. 1a. The path of a fundamental particle in the z space is obtained by rotating the hyperbola about the z_0 axis through some angle ρ. In Fig. 1b, this is the hyperbola in the plane p. We obtain the world-line of the fundamental particle in the tx plane by projection from the pole $S\equiv(0, 0, -1)$ onto the plane $P\cdot\equiv\cdot z_4=1$. In the figure, the points of the hyperbolas in the planes p and P, which are related by projection from S, are denoted by the same letter, lower case letters being used for points in p and capital letters for points in P. It should be noticed that the branch abc of the hyperbola in p is projected into the finite arc ABC in P.

Before obtaining the algebraic expressions for the motion of fundamental particles, the im-

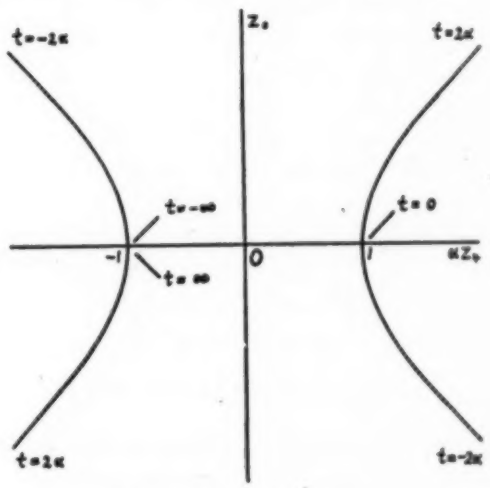

FIG. 1a. t axis in z space.

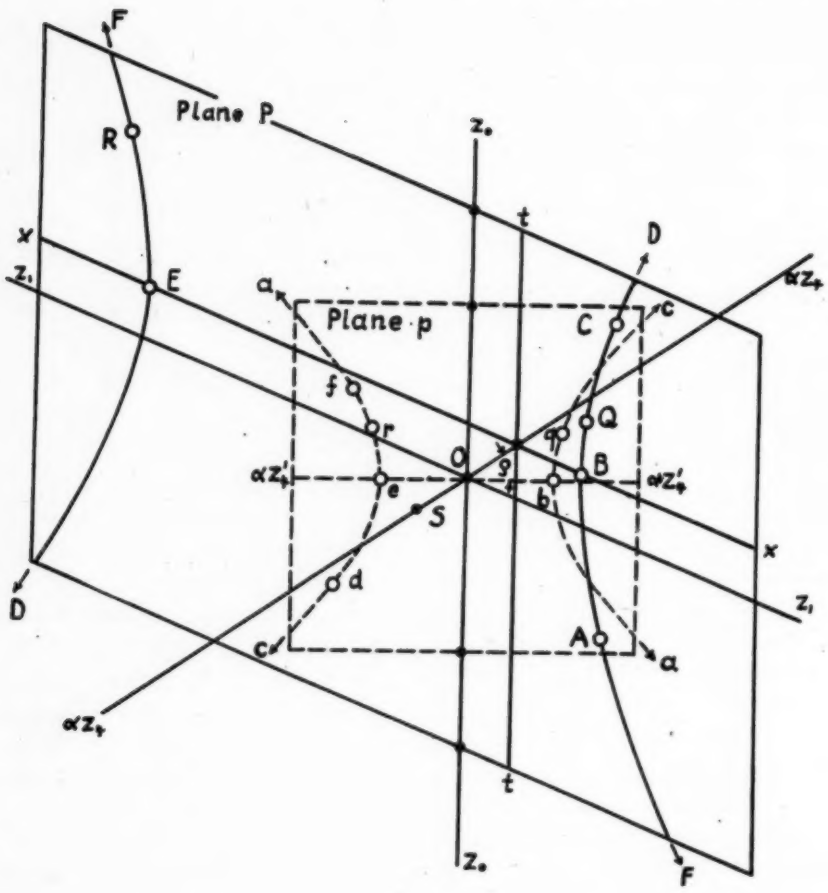

FIG. 1b. The hyperbola in plane p is the world-line of a fundamental particle in z space. The world-line in the tx plane is obtained by projection from S onto the tx plane P. Points related by this projection are denoted by the same letter (lower case letters for points in p, capitals for points in P).

portant problem of connectivity has to be dealt with. It follows immediately from the fact that an operation of projection has been used that the "ends at infinity" of any straight line must be identified. Thus both the z and x spaces have the connectivity of projective spaces. We shall, however, postulate a further connectivity. Returning for the moment to the full five-dimensional space, the points $(z_0, z_1, z_2, z_3, z_4)$ and $(z_0, -z_1, -z_2, -z_3, -z_4)$ on the hyperquadric (1.3) will be identified. This is permissible as the transformation $(z_0, z_1, z_2, z_3, z_4) \rightarrow (z_0, -z_1, -z_2, -z_3, -z_4)$ leaves the form (1.32) invariant, f being a function of z_0 only. Thus the three-dimensional section

$$z_0 = \text{constant},$$
$$z_1^2+z_2^2+z_3^2+\alpha^2 z_4^2 = z_0^2+\alpha^2 \tag{2.31}$$

is a hypersphere on which antipodal points are identified, i.e., it is an elliptic three-space. In Fig. 1b, the points A, B, Q (and a, b, q) are identified with the points C, E, R (and c, e, r), respectively. The path of a fundamental particle is the branch abc $(c \equiv a)$ of the hyperbola in p or the arc ABC $(C \equiv A)$ in P. The set of all fundamental particles is obtained by letting ρ assume all values from $-\pi/2$ to $\pi/2$.

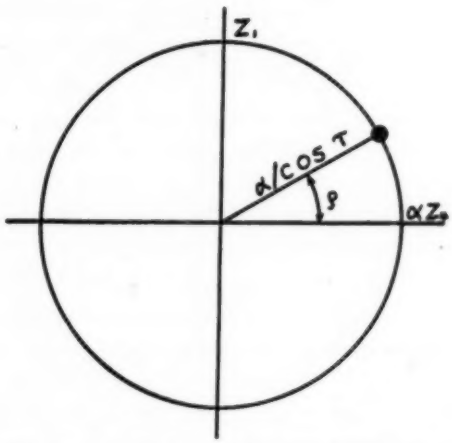

FIG. 1c. Section $z_0 = \text{constant}$ of hyperboloid $-z_0^2+z_1^2+\alpha^2 z_4^2 = \alpha^2$.

Proceeding now to the analytic treatment of the motion of fundamental particles, we put

$$X=(1/2\alpha)(t+x), \quad Y=(1/2\alpha)(t-x) \tag{2.32}$$

and have, by (1.31),

$$\begin{aligned} z_0/\alpha &= (X+Y)/(1-XY), \\ z_1/\alpha &= (X-Y)/(1-XY), \\ z_4 &= (1+XY)/(1-XY). \end{aligned} \tag{2.33}$$

Let

$$X=\tan\xi, \quad Y=\tan\eta, \tag{2.34}$$

and

$$\xi+\eta=\tau, \quad \xi-\eta=\rho; \tag{2.35}$$

then

$$\begin{aligned} &z_0/\alpha=\tan\tau, \quad z_1/\alpha=\sin\rho/\cos\tau, \\ &z_4=\cos\rho/\cos\tau. \end{aligned} \tag{2.36}$$

τ and ρ are parameters on the hyperboloid obtained by rotation of the hyperbola in Fig. 1a about the z_0 axis, ρ being, as before, the angle of rotation; $\alpha/\cos\tau$ is the distance of a point on the hyperboloid from the z_0 axis. Fig. 1c shows a section $z_0=$constant, and the geometric content of the parameters τ and ρ is clearly indicated. The elliptic connectivity identifies the pairs of points $(\tau, \rho)\equiv(\tau+\pi, \rho)$ and $(\tau, \rho)\equiv(\tau, \rho+\pi)$, as is immediately verified by (2.36). Thus we may restrict both parameters to the interval $-\pi/2$ to $\pi/2$. A straightforward calculation shows that the pair of points (X, Y) and $(-1/X, -1/Y)$ must be identified, i.e.,

$$(t, x)\equiv(-4\alpha^2 t/a, 4\alpha^2 x/a), \tag{2.37}$$

points connected by an inversion which leaves the line element (1.4, $K=1/\alpha^2$) invariant, and which changes (z_0, z_1, z_4) into $(z_0, -z_1, -z_4)$.

The equation of motion of a fundamental particle is simply $\rho=$constant, or

$$\begin{aligned} (1/2\alpha)(t+x) &= \tan\tfrac{1}{2}(\tau+\rho), \\ (1/2\alpha)(t-x) &= \tan\tfrac{1}{2}(\tau-\rho), \end{aligned} \tag{2.4}$$

where τ is a variable parameter. Eliminating τ, these equations become

$$t^2-x^2-(4\alpha/v)x+4\alpha^2=0, \quad v=\tan\rho. \tag{2.41}$$

Thus the world-lines of fundamental particles are rectangular hyperbolas which do not meet the t axis; their common transverse axis is the x axis and their asymptotes are the null lines $t=\pm(x+2\alpha/v)$.

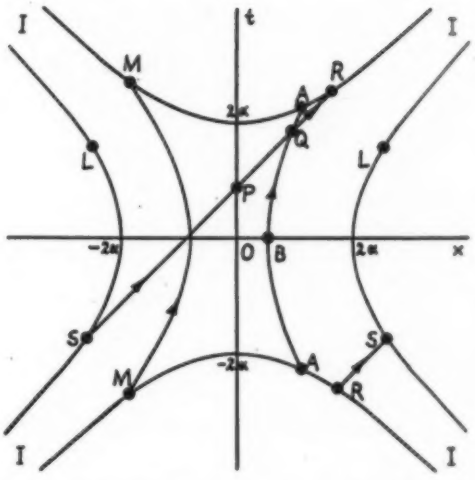

FIG. 1d. The world-lines of a fundamental particle ($ABQA$) and of a light ray (PQR, RS, SP) in elliptic universes of type I.

Putting $\rho=\pm\frac{1}{2}\pi$ in (2.4) while τ varies and $\tau=\pm\frac{1}{2}\pi$ while ρ varies, we obtain the two rectangular hyperbolas

$$x^2-t^2=\pm 4\alpha^2, \tag{2.42}$$

which bound our elliptic universe, as shown in Fig. 1d. The points outside the concave quadrangle formed by the four branches of the hyperbolas (2.42) are each identified with an interior point by the elliptic connectivity (2.37). In Fig. 1d the same letter is used to denote a pair of points which are thus identified. The topology of the tx plane is that of a torus which can be obtained by first folding the plane along the t axis, say, and connecting the branches of the hyperbolas (2.42) which meet the x axis; then folding along the x axis and connecting the two branches of the other hyperbola with each other without crossing.

Returning to the motion of a fundamental particle, characterized by the world-line $ABQA$, say, in Fig. 1d, the following simple conclusions are immediate: The fundamental particle, starting from A, moves toward the spatial origin $x=0$ with decreasing velocity until it is at instantaneous rest at B, i.e., when $t=0$ and

$$x=x_{\min}=(2\alpha/v)[+(1+v^2)^{\frac{1}{2}}-1]. \tag{2.43}$$

It then recedes with increasing velocity, first to Q, then to A, and the cycle is repeated. Thus, the observer at the spatial origin sees the funda-

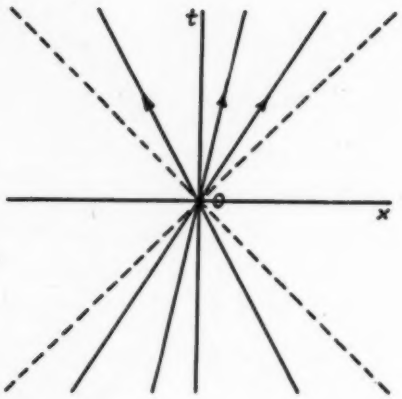

FIG. 2. Fundamental particles in universes of type II′.

mental particles move back and forth; and we may well typify this motion by the adjective *oscillating*.

The first bounding hyperbola $x^2-t^2=4\alpha^2$, whose two branches are identified, is the world-line of the fundamental particle at the greatest distance from the spatial origin. For this particle $x_{\min}=2\alpha$. As is clear from Fig. 1d, the coordinate distance 4α may be described as the "perimeter of the universe" at time $t=0$; it is the finite length of the x axis.

The second bounding hyperbola $x^2-t^2=-4\alpha^2$, whose two branches are identified, is the locus of the points at the greatest distance from the spatial origin on each world-line of a fundamental particle. To put it differently, it is the locus of the events where the fundamental particles reach the amplitudes of their oscillations, stop receding, and start moving toward the spatial origin. The coordinate time interval 4α is the period of the oscillating universe as determined by the fundamental observer $x=0$.

The periodicity of the oscillating universe is further exemplified by following the path of a light ray emitted at some event P in the direction of the positive x axis. Without loss of generality, we may take P on the t axis of our coordinate system. Keeping the elliptic connectivity in mind, we see that the world-line is PQR, RS, SP, as shown in Fig. 1d. Thus the light emitted by a fundamental observer returns to him after a finite coordinate time 4α.

It is important to note that the *physical behavior* of a cosmological model of type I is not necessarily of a periodic nature. The finite t interval $(-2\alpha, 2\alpha)$ of one period's duration may correspond to an infinite proper time. Then, since all experiences of any observer are within this finite t interval, the motion of fundamental particles is no longer oscillating, and the light-signal emitted by a fundamental observer does not return throughout his entire proper life. By Table I (case I), the proper time corresponding to the t interval $(-2\alpha, 2\alpha)$ is

$$S=\int_{-2\alpha}^{2\alpha} ds=\int_{-2\alpha}^{2\alpha}(1-t^2/4\alpha^2)^{-1} \times f^{\frac{1}{2}}(t/(1-t^2/4\alpha^2))dt. \quad (2.44)$$

We have the following criterion: A universe of type I is periodic or non-periodic in its physical behavior according as the integral S converges or diverges. Keeping this in mind, we shall nevertheless, for reasons of economy, retain the term *oscillating* to typify the general universe I and the motion of fundamental particles.

Two simple examples follow, the first of a periodic and the second of a non-periodic universe:

$$ds^2=\{(1-a/4\alpha^2)^2+t^2/\alpha^2\}^{-1}\eta_{ij}dx^i dx^j \quad (2.45)$$

is the line element of a cosmological model of type I, which will later (in part II) be identified with the Einstein universe. We immediately have

$$S=\int_{-2\alpha}^{2\alpha} dt/(1+t^2/4\alpha^2)=\pi\alpha. \quad (2.46)$$

Thus S is finite and the universe is periodic.

Next, we consider the De Sitter universe of type I, sometimes referred to as the De Sitter-Lanczos universe (see Part II). Its line element is

$$ds^2=(1-a/4\alpha^2)^{-2}\eta_{ij}dx^i dx^j. \quad (2.47)$$

In this case

$$S=\int_{-2\alpha}^{2\alpha} dt/(1-t^2/4\alpha^2) \quad (2.48)$$

TABLE II. The motion of fundamental particles.

Case	Equation of motion	Type of motion
I	$t^2-x^2-(4\alpha/v)x+4\alpha^2=0$	Oscillating
II	$t^2-x^2+(4\alpha/v)x-4\alpha^2=0$	Converging-diverging
II′	$x=vt$	
III	$x=$constant	Rest
III′	$t^2-x^2-x/v=0$	

converges logarithmically at both limits. Thus this universe is non-periodic in its physical behavior.

In conclusion, we may note that the *spherical* universe in which antipodal points of the section (2.31) are not identified is similar in its behavior to the *elliptic* universe described above, and need not be discussed in detail.

Case II—$K=-1/\alpha^2<0$.—We shall discuss this case briefly, mainly for the sake of completeness; in most applications, II′ rather than II will be appealed to. The equations of motion of fundamental particles are immediately obtained from those in Case I by applying the transformation $\alpha\rightarrow -i\alpha$. However, the variables X, Y, ξ, η, τ, ρ, introduced above, are now imaginary. In order to avoid this, the definitions will be slightly changed and these variables, in Eqs. (2.32) to (2.36), replaced by iX, iY, $i\xi$, $i\eta$, $i\tau$, $i\rho$. We then have

$$X=(1/2\alpha)(t+x), \quad Y=(1/2\alpha)(t-x), \tag{2.5}$$

$$z_0/\alpha=(X+Y)/(1+XY),$$

$$z_1/\alpha=(X-Y)/(1+XY), \tag{2.51}$$

$$z_4=(1-XY)/(1+XY),$$

$$X=\tanh\xi, \qquad Y=\tanh\eta, \tag{2.52}$$

$$\xi+\eta=\tau, \qquad \xi-\eta=\rho, \tag{2.53}$$

$$z_0/\alpha=\tanh\tau, \quad z_1/\alpha=\sinh\rho/\cosh\tau, \quad z_4=\cosh\rho/\cosh\tau, \tag{2.54}$$

where all variables are now real.

The equation of motion of a fundamental particle is $\rho=$constant, or,

$$t^2-x^2+(4\alpha/v)x-4\alpha^2=0, \quad v=\tanh\rho. \tag{2.55}$$

It should be noted that the parameter v is now limited to the interval -1 to 1. The world-lines (2.55) are rectangular hyperbolas meeting the t axis in the two fixed points $t=\pm 2\alpha$.

In contrast to the previous case, the only connectivity of the tx plane in the present case is that of a projective plane. All finite points represent distinct events and no two are identified.

Rather than continue the examination of this cosmological model in the c.c.s. II we shall find it more convenient now to introduce the c.c.s. II′.

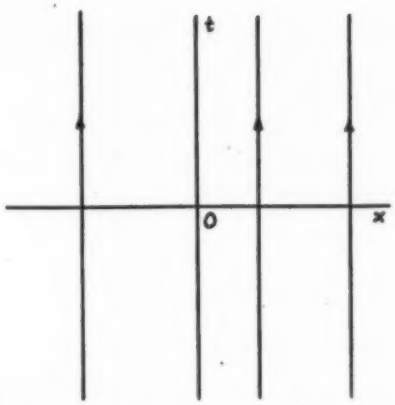

FIG. 3. Fundamental particles in universes of type III.

Case II′.—From the discussion immediately preceding equation (1.5) we know that it is the group of Lorentz transformations which leaves the line element II′ invariant. Under such transformations the t axis is carried into the line

$$x=vt, \quad |v|<1, \tag{2.6}$$

which is the world-line of a fundamental particle.

The fundamental particles all move radially, each with constant velocity v. They *converge*, meet at the origin $t=x=0$, and then *diverge*. This simple behavior of the particles is shown in Fig. 2. It should be noted that the whole observable universe, into which particles can penetrate, is confined to the interior and the surface of the double light-cone with vertex at the origin $t=x=0$.

The explicit transformation which leads from the coordinate system II to II′ is easily obtained in terms of the variables X, Y. As we saw in the first section, the transformation, in the z space, is

$$(z_0, z_1, \alpha z_4)\rightarrow(-\alpha z_4, z_1, z_0). \tag{2.61}$$

The minus sign has to be introduced in order that the sense of time be preserved. Using (2.51), we obtain, by a short calculation,

$$(X, Y)\rightarrow((X-1)/(X+1), \quad (Y-1)/(Y+1)). \tag{2.62}$$

If this transformation is applied to the equation of motion (2.55), the equation of motion (2.6) results. Thus the parameter v is actually the same in both (2.55) and (2.6). It might be

noted that (2.62) transforms the time interval $(-2\alpha, 2\alpha)$ in Case II into the interval $(0, \infty)$ of the t axis in II′.

Case III—$K=0$.—It follows from the transformation (1.42) that the equation of motion of fundamental particles is

$$x=\text{constant}. \tag{2.7}$$

The fundamental particles are at *rest*, as shown in Fig. 3.

Case III′.—Subjecting equation (2.7) to the inversion (1.61), we obtain the equation of motion

$$t^2-x^2-x/v=0, \tag{2.8}$$

where v is a constant. Thus the motion of fundamental particles is along rectangular hyperbolas, all touching the t axis at the origin.

The results of this section are summarized in Table II.

APPENDIX

In this section the theory of continuous groups will be applied to the problem of finding all functions $\gamma(t, r)$ in the line element (1.1) which satisfy the postulate of homogeneity. This principle states that the space with the metric form (1.1) admits a continuous group G of motions which do not move the t axis into itself. The contravariant components ξ^i of the infinitesimal motion, which generates the group, satisfy the equations of Killing[10]

$$\xi^k g_{ij,k}+g_{ik}\xi^k{}_{,j}+g_{jk}\xi^k{}_{,i}=0, \tag{A1}$$

where the comma denotes partial differentiation; thus $g_{ij,k}=\partial g_{ij}/\partial x^k$, etc.

After solving Killing's equations for the ξ^i, the finite transformations of a one-parameter group are obtained by integrating the system of differential equations[11]

$$d\bar{x}^i/dp=\xi^i(\bar{x}^0, \bar{x}^1, \bar{x}^2, \bar{x}^3), \tag{A11}$$

with the initial conditions $\bar{x}^i=x^i$ when $p=0$; p is the parameter of the group.

It follows from the isotropy of the space that it is sufficient to find a one-parameter subgroup G_1 of the complete group G, such that G_1 moves a plane through the t axis into itself. The transformations of G_1 may be combined with suitable spatial rotations to yield the required three-parameter group G. The invariant plane of G_1, the tx plane, say, determines a preferred spatial direction. Thus it is convenient to introduce cylindrical spatial coordinates (x, ρ, φ) in which the line element (1.1) becomes

$$ds^2=\gamma(t, r)(dt^2-dx^2-d\rho^2-\rho^2 d\varphi^2); \qquad r^2=x^2+\rho^2. \tag{A2}$$

As we are not interested in spatial rotations, we may at the outset assume

$$\bar{\varphi}=\varphi, \quad \xi^\varphi=0. \tag{A21}$$

The equations of Killing are now, for $i=j$,

$$\begin{aligned}
&\gamma_{,t}\xi^t+\gamma_{,r}\xi^x x/r+\gamma_{,r}\xi^\rho\rho/r+2\gamma\xi^t{}_{,t}=0,\\
&\gamma_{,t}\xi^t+\gamma_{,r}\xi^x x/r+\gamma_{,r}\xi^\rho\rho/r+2\gamma\xi^x{}_{,x}=0,\\
&\gamma_{,t}\xi^t+\gamma_{,r}\xi^x x/r+\gamma_{,r}\xi^\rho\rho/r+2\gamma\xi^\rho{}_{,\rho}=0,\\
&\gamma_{,t}\xi^t+\gamma_{,r}\xi^x x/r+\gamma_{,r}\xi^\rho\rho/r+2\gamma\xi^\rho/\rho=0,
\end{aligned} \tag{A22}$$

and for $i\neq j$,

$$\begin{aligned}
&\xi^t{}_{,x}-\xi^x{}_{,t}=0, \quad \xi^t{}_{,\rho}-\xi^\rho{}_{,t}=0,\\
&\xi^x{}_{,\rho}+\xi^\rho{}_{,x}=0, \quad \xi^t{}_{,\varphi}=\xi^x{}_{,\varphi}=\xi^\rho{}_{,\varphi}=0.
\end{aligned} \tag{A23}$$

The last three equations of (A23) show that all ξ^i are independent of φ. From the Eqs. (A22), we have

$$\xi^t{}_{,t}=\xi^x{}_{,x}=\xi^\rho{}_{,\rho}=\xi^\rho/\rho. \tag{A24}$$

We shall now proceed as follows: The first three equations of (A23) and Eqs. (A24) will be solved and the functions ξ^i obtained. Substituting in any of the Eqs. (A22) we next obtain a differential equation for γ. This equation will be solved under different assumptions which exhaust all possibilities. Finally, substituting for ξ^i in Eqs. (A11), the finite transformations of the group G_1 may be obtained. However, as the main purpose of this appendix is to determine all admissible functions γ, this last step will not be discussed in detail and only the results will be given.

From the last of the relations (A24), we have

$$\xi^\rho=\rho f(t, x); \quad \xi^\rho{}_{,\rho\rho}=0.$$

[10] L. P. Eisenhart, *Riemannian Geometry* (Princeton University Press, Princeton, 1926), Eq. (70.1).
[11] Reference 10, Eq. (66.3).

But, by (A24) and (A23),

$$\xi^{\rho}{}_{,\rho\rho}=\xi^{t}{}_{,t\rho}=\xi^{t}{}_{,\rho t}=\xi^{\rho}{}_{,tt}.$$

Thus $\xi^{\rho}{}_{,tt}=0$ and, similarly, $\xi^{\rho}{}_{,xx}=0$. Therefore $f_{,tt}=f_{,xx}=0$ and

$$\xi^{\rho}=\rho(axt+2bx+2ct+d),$$

where a, b, c, d, are constants. From this and (A23), (A24), we have

$$\xi^{t}{}_{,t}=\xi^{\rho}{}_{,\rho}=axt+2bx+2ct+d,$$

$$\xi^{t}{}_{,\rho}=\xi^{\rho}{}_{,t}=ax\rho+2c\rho.$$

Therefore

$$\xi^{t}=\tfrac{1}{2}ax(t^2+\rho^2)+c(t^2+\rho^2)+2btx+dt+F(x),$$

and, similarly,

$$\xi^{x}=\tfrac{1}{2}at(x^2-\rho^2)+b(x^2-\rho^2)+2ctx+dx+\Phi(t).$$

But $\xi^{t}{}_{,x}=\xi^{x}{}_{,t}$, by (A23), and thus

$$\tfrac{1}{2}a(t^2+\rho^2)+2bt+F'(x)=\tfrac{1}{2}a(x^2-\rho^2)+2cx+\Phi'(t).$$

Hence $a=0$, $F=cx^2+ex+f$, $\Phi=bt^2+et+g$, e, f, g, being constants. Finally, we obtain

$$\xi^{t}=c(t^2+x^2+\rho^2)+2btx+dt+ex+f, \quad \text{(A25)}$$

$$\xi^{x}=b(t^2+x^2-\rho^2)+2ctx+et+dx+g, \quad \text{(A26)}$$

$$\xi^{\rho}=\rho(2bx+2ct+d). \quad \text{(A27)}$$

Substituting in any of the Eqs. (A22), we have, on multiplication by r,

$$r\gamma_{,t}(ct^2+cr^2+dt+f)+r^2\gamma_{,r}(2ct+d)$$
$$+2r\gamma(2ct+d)=-x\{r\gamma_{,t}(2bt+e)$$
$$+\gamma_{,r}(bt^2+br^2+et+g)+4br\gamma\}. \quad \text{(A28)}$$

As γ is a function of t and r only, we have the alternative $c=d=f=0$ or $b=e=g=0$. The latter possibility does not lead to suitable transformations: Using (A11), a short calculation yields the explicit form of the finite transformations and it is seen that $r=0$ implies $\bar{r}=0$, i.e., the t axis is moved into itself. We therefore put $c=d=f=0$ and have

$$r\gamma_{,t}(2bt+e)+\gamma_{,r}(bt^2+br^2+et+g)$$
$$+4br\gamma=0. \quad \text{(A3)}$$

This is the differential equation for γ. The transformations of the group G_1 are obtained by solving the system of equations

$$d\bar{t}/dp=\bar{x}(2b\bar{t}+e), \quad \text{(A31)}$$

$$d\bar{x}/dp=b(\bar{t}^2+\bar{x}^2-\bar{\rho}^2)+e\bar{t}+g, \quad \text{(A32)}$$

$$d\bar{\rho}/dp=2b\bar{x}\bar{\rho}, \quad \text{(A33)}$$

with the initial conditions $(\bar{t}, \bar{x}, \bar{\rho})=(t, x, \rho)$ when $p=0$.

We now have the following mutually exclusive possibilities:

1. $b=e=0$, $g\neq 0$.—Then $\gamma_{,r}=0$; thus

$$\gamma=\gamma(t). \quad \text{(A4)}$$

This is the universe III, Table I. Solving (A31) to (A33), we immediately find that the transformations of the group G_1 are translations along the x axis.

2. $b=0$, $e\neq 0$.—Making a suitable choice of the temporal origin, we replace $t+g/e$ by t. Then (A3) simplifies to

$$r\gamma_{,t}+t\gamma_{,r}=0.$$

Thus

$$\gamma=\gamma(t^2-r^2). \quad \text{(A5)}$$

This is the universe II′, Table I. The finite transformations of G_1 are easily obtained and seen to be Lorentz transformations in the variables t and x.

3. $b\neq 0$, $g/b-e^2/4b^2\neq 0$.—Changing the temporal origin, we replace $t+e/2b$ by t and also write $K/4$ for $(g/b-e^2/4b^2)^{-1}$. Then (A3) becomes

$$2tr\gamma_{,t}+\gamma_{,r}(t^2+r^2+4/K)+4r\gamma=0. \quad \text{(A61)}$$

Introducing

$$v=t/(1-(t^2-r^2)K/4),$$

and putting

$$\gamma=f(t, v)v^2/t^2,$$

a short computation shows that (A61) simplifies to $f_{,t}=0$. Therefore $f=f(v)$, and we have

$$\gamma=(1-(t^2-r^2)K/4)^{-2}$$
$$\times f(t/(1-(t^2-r^2)K/4)). \quad \text{(A6)}$$

This is the universe I or II, Table I, according as $K>0$ or $K<0$. The transformations which

TABLE III. Line elements and motion.

Type	K	γ	Equation of motion	Type of motion
I	4	$(1-a)^{-2}f(t/(1-a))$	$t^2-r^2-(2/v)r+1=0$	Oscillating
II	-4	$(1+a)^{-2}f(t/(1+a))$	$t^2-r^2+(2/v)r-1=0$	
II′		$\gamma(a)$	$r=vt$	Converging-diverging
III	0	$\gamma(t)$	$r=$constant	Rest
III′		$(1/a^2)f(-t/a)$	$t^2-r^2-(1/v)r=0$	

leave the form (A6) invariant are

$$\bar{x}\pm(\bar{t}^2-\bar{\rho}^2)^{\frac{1}{2}}=2K^{-\frac{1}{2}}\tan\{\tan^{-1}\tfrac{1}{2}K^{\frac{1}{2}}\times(x\pm(t^2-\rho^2)^{\frac{1}{2}})+2K^{-\frac{1}{2}}bp\},\quad(A62)$$

$$\bar{\rho}/\bar{t}=\rho/t.$$

4. $b\neq 0$, $g/b-e^2/4b^2=0$.—This is the last possibility and concludes our examination of spaces which admit a group G of motions. From Eq. (A61), we now have

$$2rt\gamma_{,t}+\gamma_{,r}(t^2+r^2)+4r\gamma=0.\quad(A71)$$

Putting

$$v=-t/(t^2-r^2),\quad \gamma=f(t,v)v^2/t^2,$$

we obtain $f_{,t}=0$. Thus $f=f(v)$, and

$$\gamma=(t^2-r^2)^{-2}f(-t/(t^2-r^2)).\quad(A7)$$

This is the universe III′, Table I. The finite transformations which leave the line element (A7) invariant can be shown to be

$$\bar{x}\pm(\bar{t}^2-\bar{\rho}^2)^{\frac{1}{2}}=(x\pm(t^2-\rho^2)^{\frac{1}{2}}-bp)^{-1},\qquad \bar{\rho}/\bar{t}=\rho/t.\quad(A72)$$

In each of the cases examined here the transformations of the group G can easily be shown to agree with those discussed in Section 1.

II. Special Problems and Applications

3. INTRODUCTION

We give here a short summary of the main results of Part I[12] of this paper before proceeding to applications.

In Part I, the metric forms and the associated motion of fundamental particles (nebulae), suitable for the kinematical description of our universe at large, are examined on the basis of three postulates, namely, the constancy of the velocity of light, spatial isotropy, and homogeneity. The permissible line elements are of the form

$$ds^2=\gamma(t,r)(dt^2-dx^2-dy^2-dz^2)=\gamma(t,r)\eta_{ij}dx^idx^j,\quad r^2=x^2+y^2+z^2.\quad(3.1)$$

The function γ is restricted by the homogeneity requirement, and three distinct types of cosmological models are obtained. The permissible functions γ and the radial equations of motion of fundamental particles are listed in Tables I and II.

[12] The numbering of sections, equations, and footnotes is carried over from Part I. References to Eqs. (0.1)–(2.8) and to footnotes 1–11 are to those of Part I.

We introduce two changes of notation. The equations of motion of fundamental particles, which move radially, will be written in terms of t and r; it will be understood that they are to be supplemented by the equations θ, $\varphi=$constant. Next, in cases I and II, we change the coordinate unit, measuring t and r in terms of the *natural cosmological unit* 2α. This is achieved by the coordinate transformation

$$x^i/2\alpha\rightarrow x^i.\quad(3.2)$$

The coordinates, being measured in natural units, now have no physical dimensions. This introduces a considerable simplification in most formulas. Table III is a summary of the main results of Sections 1 and 2.

In Table III, v is a parameter characterizing the individual fundamental particle, and

$$a=t^2-r^2.\quad(3.3)$$

In universes of type I, the points

$$(t,r)\equiv(-t/a,r/a)\quad(3.4)$$

are identified by virtue of the elliptic connectivity.

The forms II′ are obtained from II by the coordinate transformation

$$(X, Y)\rightarrow((X-1)/(X+1), (Y-1)/(Y+1)), \quad (3.5)$$

where

$$X=t+r, \quad Y=t-r. \quad (3.6)$$

The forms III′ are obtained from III by the inversion

$$(t, r)\rightarrow(-t/a, r/a). \quad (3.7)$$

Using spherical spatial coordinates (r, θ, φ), the transformation equations (3.5) and (3.7) must be supplemented by

$$(\theta, \varphi)\rightarrow(\theta, \varphi). \quad (3.8)$$

4. THE COSMOLOGICAL AND THE ROBERTSON COORDINATE SYSTEMS

The coordinate system by which cosmological models have usually been described in the past is, in its general form, due to H. P. Robertson.[13] An *a priori* separation of space-time into space and time is ensured by writing the line element in the form

$$ds^2=d\tau'^2-R_1^2d\sigma^2, \quad (4.1)$$

where $d\sigma^2$ is the positive definite metric of a 3-space. The assumptions of isotropy and homogeneity of space impose two restrictions: $R_1=R_1(\tau')$ is a function of time only, and $d\sigma^2$ is the differential form of a 3-space of constant curvature $k=1$, -1, or 0.

Let

$$\tau=\int d\tau'/R_1(\tau'), \quad R(\tau)=R_1(\tau'). \quad (4.11)$$

Then the line element (4.1) assumes the form

$$ds^2=R^2(\tau)(d\tau^2-d\sigma^2), \quad (4.2)$$

where, with suitable spatial coordinates, we may write

$$d\sigma^2=d\rho^2+\sin^2\rho(d\theta^2+\sin^2\theta d\varphi^2), \quad \text{if } k=1, \quad (4.21)$$

$$d\sigma^2=d\rho^2+\sinh^2\rho(d\theta^2+\sin^2\theta d\varphi^2), \quad \text{if } k=-1, \quad (4.22)$$

$$d\sigma^2=d\rho^2+\rho^2(d\theta^2+\sin^2\theta d\varphi^2), \quad \text{if } k=0. \quad (4.23)$$

[13] H. P. Robertson "On the Foundations of Relativistic Cosmology," Proc. Nat. Acad. Sci. 15, 822–829 (1929).

The cosmological line element (3.1), in spatial polar coordinates, is

$$ds^2=\gamma(t, r)(dt^2-dr^2-r^2d\theta^2-r^2\sin^2\theta d\varphi^2), \quad (4.3)$$

where all permissible functions γ are listed in Table III. It will be shown that the line elements (4.2) and (4.3) are equivalent and that they can be obtained from each other by a coordinate transformation.

We introduce the variables:

$$X=(t+r), \quad Y=(t-r), \quad (4.31)$$

and

$$\xi=\tfrac{1}{2}(\tau+\rho), \quad \eta=\tfrac{1}{2}(\tau-\rho). \quad (4.32)$$

It may be noted that X, Y, and, as will be seen shortly, ξ, η, τ, ρ, are essentially the same quantities as were introduced in Section 2 (where y and z were both zero and therefore x appeared instead of r).

Case I—$K=4$.—In terms of the variables X and Y, we have, by (4.3) and Table III,

$$ds^2=f\left(\frac{1}{2}\frac{X+Y}{1-XY}\right)\cdot\frac{1}{4}\left\{1+\left(\frac{X+Y}{1-XY}\right)^2\right\}\cdot\frac{4dXdY-(X-Y)^2(d\theta^2+\sin^2\theta d\varphi^2)}{(1+X^2)(1+Y^2)}.$$

The transformation to the Robertson coordinate system is

$$X=\tan\xi, \quad Y=\tan\eta. \quad (4.4)$$

We immediately obtain

$$ds^2=f(\tfrac{1}{2}\tan(\xi+\eta))\cdot\tfrac{1}{4}\sec^2(\xi+\eta)\cdot\{4d\xi d\eta-\sin^2(\xi-\eta)(d\theta^2+\sin^2\theta d\varphi^2)\},$$

which, by (4.32), assumes the form (4.2–4.21), where

$$R^2(\tau)=\tfrac{1}{4}\sec^2\tau f(\tfrac{1}{2}\tan\tau), \quad \text{and} \quad k=1. \quad (4.41)$$

From this last relation we note that not all Robertson universes, with $k=1$, can actually be brought into the cosmological form I. Only if the function $R(\tau)$ is periodic and of period π is this possible. This can also be seen, and perhaps better, from the transformations (4.4). An increase of π in τ changes, by (4.32), both ξ and η by an amount $\frac{1}{2}\pi$. The effect of this is to transform X, Y into $-1/X$, $-1/Y$; and events (X, Y) and $(-1/X, -1/Y)$ are identified in the elliptic connectivity, given by Eq. (3.4).

Thus the forms (4.2–4.21) seem to include a more extensive class of cosmological models than the corresponding forms I, Table III. We shall show, however, that this limitation is not very serious.

First, the physical significance of the τ-interval π will be examined. If (4.21) is interpreted as the line element of an elliptic 3-space, the finite length of a straight line is π, the linear unit being, as usually in elliptic geometry, the radius of curvature $1/k$. Equations (4.2–4.21) show that the coordinate velocity of light rays traveling radially is 1. Thus the interval π on the τ scale is, physically, the time interval in which light circumnavigates the universe and returns to the fundamental particle which emitted it. This is, obviously, a considerable time interval and so, even in a case where $R(\tau)$ is not periodic, a large portion of the Robertson universe can always be represented by our model I.

Next, it should be noted that the periodicity of $R(\tau)$ does not necessarily imply the periodicity of $R_1(\tau')$. In fact, in some important special models the infinite proper interval $(-\infty, \infty)$ on the τ' scale transforms into a finite interval π on the τ-scale, in which case the universe of type I is non-periodic in its physical behavior (see discussion in Section 2).

To illustrate this, we examine the De Sitter-Lanczos universe,[14] where $k=1$, and

$$R_1^2(\tau')=\tfrac{1}{4}\cosh^2 2(\tau'-\tau_0'), \qquad (4.42)$$

τ_0' being a constant. $R_1(\tau')$ is not periodic. We have

$$\tau=2\int \operatorname{sech} 2(\tau'-\tau_0')d\tau' = 2\tan^{-1} e^{2(\tau'-\tau_0')}-\tfrac{1}{2}\pi. \qquad (4.43)$$

The interval $-\infty<\tau'<\infty$ corresponds to $-\frac{1}{2}\pi<\tau<\frac{1}{2}\pi$ and, by (4.4), to $-1<t<1$. A short calculation yields

$$R^2(\tau)=\tfrac{1}{4}\sec^2\tau. \qquad (4.44)$$

Therefore, by (4.41), $f=1$, and, by I, Table III, the De Sitter-Lanczos line element has the form

$$ds^2=(1-a)^{-2}\eta_{ij}dx^i dx^j \qquad (4.45)$$

in the c.c.s.

[14] H. P. Robertson, "Relativistic Cosmology," Rev. Mod. Phys. 5, 62–90 (1933), Eq. (6.6), where we put $c=1$, $a=\frac{1}{4}$.

We have shown that there is no essential *metrical* difference between Robertson's universes and ours. However, there is an essential *topological* difference. Suppressing θ and φ, we have in the Robertson case universes represented by a (τ, ρ) plane in which the events (τ, ρ) and $(\tau, \rho+\pi)$ are identified; i.e., the universes are topologically equivalent to a cylinder, whose τ dimension goes to infinity. The topology of the universe I is different; the events (τ, ρ) and $(\tau+\pi, \rho)$ must also be identified, because in the c.c.s. they are connected by the inversion (3.4). Thus we now have the topology, *not* of a *cylinder*, but of a *torus*. Even in the case of a "spherical universe," the topology remains that of a torus as the event (τ, ρ) must now be identified with $(\tau, \rho+2\pi)$ and with $(\tau+2\pi, \rho)$. Though the topological difference between Robertson's universes and ours does not reveal itself in the problems considered in this paper, it will become decisive in the treatment of Maxwell's and Dirac's equations on a cosmological background.

Case II—$K=-4$.—The transformation to the Robertson coordinate system is obtained from (4.4) by replacing X, Y, ξ, η, by iX, iY, $i\xi$, $i\eta$:

$$X=\tanh\xi, \quad Y=\tanh\eta. \qquad (4.5)$$

Corresponding to (4.41), we now have

$$R^2(\tau)=\tfrac{1}{4}\operatorname{sech}^2\tau\cdot f(\tfrac{1}{2}\tanh\tau),$$

and

$$k=-1. \qquad (4.51)$$

There are in this case no limitations on the function $R(\tau)$.

Case II'.—The line element is, by (4.3) and Table III,

$$ds^2=\gamma(XY)\cdot XY\cdot\frac{dXdY-(\frac{1}{2}(X-Y))^2(d\theta^2+\sin^2\theta d\varphi^2)}{XY}.$$

The transformation to the Robertson coordinate system[15] is

$$X=e^{2\xi}, \quad Y=e^{2\eta}. \qquad (4.6)$$

[15] This transformation has been obtained by A. G. Walker, "On the Formal Comparison of Milne's Kinematical System with the Systems of General Relativity," M. N. R. A. S. 95, 263–269 (1935).

We immediately obtain

$$ds^2=\gamma(e^{2(\xi+\eta)})\cdot e^{2(\xi+\eta)}$$
$$\cdot\{4d\xi d\eta-\sinh^2(\xi-\eta)(d\theta^2+\sin^2\theta d\varphi^2)\},$$

which, by (4.32), assumes the form (4.2–4.22), where

$$R^2(\tau)=e^{2\tau}\gamma(e^{2\tau}), \quad \text{and} \quad k=-1. \quad (4.61)$$

Comparing (4.5) and (4.6), we easily regain the transformation formula (3.5) leading from Case II to Case II′.

Case III, III′—K=0.—The cosmological line element III is of the form (4.2–4.23) and no transformation is required. For the sake of completeness, we may write

$$X=2\xi, \quad Y=2\eta, \quad (4.7)$$

and

$$R^2(\tau)=\gamma(\tau), \quad k=0. \quad (4.71)$$

In Case III′, the transformation is the inversion (3.7) and need not be discussed further.

It has been shown that our cosmological models are the same as those considered by Robertson and his predecessors. However, the mathematical picture as well as the physical emphasis differ considerably in the two representations. The universes which, in Robertson's coordinates, are of type $k=1, -1, 0$, are, in a c.c.s., of type I, II′ (or II), III (or III′), respectively. In the Robertson coordinate system the fundamental particles are at rest; in a c.c.s. their motion is, in the three cases, the oscillating motion, the converging-diverging motion, and rest, respectively. In Robertson's coordinates the velocity of light is a function of position and direction; in a c.c.s. it is constant.

The form of the cosmological line element (4.2) in the Robertson coordinate system suggests the following consideration. The temporal translation

$$\tau\rightarrow\tau+\tau_0 \quad (4.8)$$

changes the function $R(\tau)$, and therefore also the line element (4.2), but preserves the type of the universe, which is determined by $d\sigma^2$. The translation (4.8) induces the following transformations in the cosmological coordinates:

Case I: $(X, Y)\rightarrow((X+t_0)/(1-t_0X),$
$(Y+t_0)/(1-t_0Y))$, (4.81)

Case II′: $(t, r)\rightarrow(t_0t, t_0r)$, (4.82)

Case III: $(t, r)\rightarrow(t+t_0, r)$. (4.83)

These transformations change the cosmological line elements into others which are formally different but of the same type; the equations of motion of fundamental particles are preserved, and, in particular, the t axis is moved into itself. The event $(t_0, 0)$ is transformed into (0, 0), (1, 0), (0, 0), respectively. Thus, by a suitable choice of the line element and the function γ, any event on the t axis may be made to coincide with the natural origin (0, 0), in Cases I and III, and with (1, 0) in Case II′. This different behavior of models of type II′ is due to the fact that the motion of fundamental particles has a singularity at $t=0$ (when the universe shrinks to a point).

The magnification (4.82) may also be applied to a model of type III. The event $(t_0, 0)$ is transformed into (1, 0), if $t_0\neq0$.

5. SOME SPECIAL UNIVERSES

In this section some special cosmological models will be examined and the line element obtained in a c.c.s.

Universes of Type I.—The line elements of these universes are of the form

$$ds^2=(1-a)^{-2}f(t/(1-a))\eta_{ij}dx^idx^j, \quad (5.1)$$

and the motion of fundamental particles is, in general, the *oscillating* motion. As was seen in Section 2, some of the universes I are, physically, not of a periodic nature. In such cases the fundamental particles only approach and then recede from a fundamental observer within his infinite proper life.

The simplest model, in our representation, is the non-periodic *De Sitter-Lanczos universe*, which we denote by S_1. Its line element is

$$ds^2=(1-a)^{-2}\eta_{ij}dx^idx^j. \quad (5.11)$$

The line element of *Einstein's cylindrical universe* E is usually obtained in the Robertson coordinate system:

$$ds^2=d\tau'^2-d\sigma^2, \quad (5.12)$$

where $d\sigma^2$ is the metric (4.21) of a 3-space of constant curvature $k=1$. By (4.1) and (4.11),

TABLE IV. Special cosmological models.

Case	*	Universe		γ	Type	Transformation of line elements
$K=4$ $(k=1)$	Oscillating	S_1	(a)	$(1-a)^{-2}$	I	$(a)\rightarrow(b)$, (i), (n), (o), $(d)\rightarrow(f)$, $(e)\rightarrow(g)$, $(j)\rightarrow(k)$, by transformation $(X, Y)\rightarrow((1+X)/(1-X), (1+Y)/(1-Y))$.
			(b)	$1/4t^2$	I	
		E	(c)	$4\{(1-a)^2+4t^2\}^{-1}$	I	
$K=-4$ $(k=-1)$	Converging-diverging	M_2	(d)	1	II′	(d), $(l)\rightarrow(e)$, (m), $(n)\rightarrow(o)$, $(q)\rightarrow(r)$, by inversion $(t, r)\rightarrow(-t/a, r/a)$.
			(e)	$1/a^2$	II′	
			(f)	$4(1+a-2t)^{-2}$	II	
			(g)	$4(1+a+2t)^{-2}$	II	
		S_2	(h)	$(1-a)^{-2}$	II′	
			(i)	$1/4t^2$	II	
		Mi	(j)	$1/a$	II′	
			(k)	$4\{(1+a)^2-4t^2\}^{-1}$	II	
$K=0$ $(k=0)$	Rest	M_3	(l)	1	III	$(f)\rightarrow(m)$, (a), $(h)\rightarrow(p)$, by temporal translation $t\rightarrow t+1$.
			(m)	$1/a^2$	III′	
		S_3	(n)	$1/4t^2$	III	
			(o)	$1/4t^2$	III′	
			(p)	$(1/a^2)(1+2t/a)^{-2}$	III′	
		ES	(q)	t^4	III	$(g)\rightarrow(m)$, by temporal translation $t\rightarrow t-1$.
			(r)	t^4/a^6	III′	

* Motion of fundamental particles.

we have $R_1^2(\tau')=R^2(\tau)=1$, and Eq. (4.41) then yields

$$f(x)=4(1+4x^2)^{-1}.$$

Thus the Einstein universe has the cosmological line element

$$ds^2=4\{(1-a)^2+4t^2\}^{-1}\eta_{ij}dx^idx^j. \quad (5.13)$$

We see that in the c.c.s. the Einstein universe does not have a simple line element, and that it seems more artificial than the De Sitter model.

Finally, we examine very briefly *Lemaître's universe*. The Lemaître line element in the Robertson coordinate system[16] is of the type $k=1$, and the function $R_1(\tau')$ is defined by

$$d\tau'=(3R_1)^{\frac{1}{2}}dR_1/(R_1+2)^{\frac{1}{2}}(R_1-1).$$

Since $d\tau'=R_1d\tau$, by (4.11), and $R_1(\tau')=R(\tau)$, we immediately have

$$d\tau=3^{\frac{1}{2}}dR/(R^2+2R)^{\frac{1}{2}}(R-1),$$

and, on integration,

$$R=2/(3\coth^2\tfrac{1}{2}\tau-1). \quad (5.14)$$

Thus R is not a periodic function of τ and we can, at best, represent only a portion of the Lemaître universe in a c.c.s.

Universes of Type II, II′.—The line elements of these universes are preferably taken in the form II′:

$$ds^2=\gamma(a)\eta_{ij}dx^idx^j. \quad (5.2)$$

The fundamental particles move with constant coordinate velocities and they all meet in a point at time $t=0$. We have described this motion as *converging-diverging*.

The transition from the form II′ to II is achieved by the inverse of the transformation (3.5):

$$(X, Y)\rightarrow((1+X)/(1-X), (1+Y)/(1-Y)). \quad (5.21)$$

[16] Reference 14, Eq. (8.2), where we put $R_*=1$.

The simplest model of Type II′ is the *Minkowski universe* ($\gamma(a)=1$) which we denote by M_2. Its metric is given by the pseudo-Euclidean line element

$$ds^2=\eta_{ij}dx^i dx^j. \tag{5.22}$$

The De Sitter line element (5.11), besides being of type I, is also quite obviously of type II′. Thus among the models II′ we have a *De Sitter universe*, which we denote by S_2, and whose line element is

$$ds^2=(1-a)^{-2}\eta_{ij}dx^i dx^j. \tag{5.23}$$

Subjecting this form to the transformation (5.21), we obtain the line element of S_2 in the form II:

$$ds^2=(1/4t^2)\eta_{ij}dx^i dx^j. \tag{5.24}$$

This is also a special case of the forms (5.1) of type I, where $f(x)=1/4x^2$. Thus (5.24) is an alternative line element of the universe S_1.[17]

Finally, we mention a model which we call *Milne's universe* and denote by *Mi*. Its line element is

$$ds^2=(1/a)\eta_{ij}dx^i dx^j. \tag{5.25}$$

In this universe, the motion of free particles along any geodesic is identical with the motion of particles in Milne's "kinematical relativity" under the influence of the "substratum"[18]; in particular, the fundamental particles, which form Milne's substratum, behave like those in all our models II′. We may add that in Robertson's coordinates Milne's universe has a form analogous to Einstein's universe; its line element is (5.12) but $d\sigma^2$ is now the metric (4.22) of a 3-space of constant curvature $k=-1$.

Universes of Type III, III′.—It is convenient to use the c.c.s. III:

$$ds^2=\gamma(t)\eta_{ij}dx^i dx^j. \tag{5.3}$$

In these models the fundamental particles are at *rest.*

The pseudo-Euclidean line element (5.22), besides being of type II′, is also of type III. Thus, among the models III, we have a *Minkowski universe* M_3 with line element

$$ds^2=\eta_{ij}dx^i dx^j. \tag{5.31}$$

The De Sitter line element (5.24) is immediately seen to be a special case of the forms (5.3). Thus we have a third *De Sitter universe* S_3 of type III and with line element

$$ds^2=(1/4t^2)\eta_{ij}dx^i dx^j. \tag{5.32}$$

S_3 is sometimes referred to as the "stationary" De Sitter universe.[19]

The *Einstein-De Sitter universe ES* has, in the Robertson coordinate system,[20] a line element of type $k=0$, where

$$R_1=(3\tau')^{\frac{1}{3}}. \tag{5.33}$$

Using (4.11), a short calculation yields the line element in the c.c.s. III:

$$ds^2=t^4\eta_{ij}dx^i dx^j. \tag{5.34}$$

This cosmological model has also been proposed by *Dirac.*[21]

It must be emphasized that the two Minkowski universes M_2 and M_3 are quite distinct cosmological models and differ in their physical behavior; e.g., M_2 exhibits a nebular red shift, M_3 does not. M_2 and M_3 are characterized by the same line element (5.22) but by different motions of fundamental particles, i.e., the converging-diverging motion and rest, respectively. A corresponding statement holds for the three De Sitter universes S_1, S_2, S_3, which may be described by the same metric form (5.24) and by the three different types of motion of fundamental particles.

We conclude this section with a short examination of *stationary universes.*[22] We may define this term as follows: A universe is stationary if its line element and the equation of motion of its fundamental particles are invariant in form under a coordinate transformation which moves the t axis into itself and which transforms any non-singular event on it into any other.

[17] The transformation (5.21) is a special case of (4.81), $t_0=1$, and therefore changes any line element of type I into another of the same type, and also preserves the equation of motion of fundamental particles. A similar statement is easily verified to hold for the inversion (3.7) applied to a line element of type II′.

[18] C. Gilbert, "On the Occurrence of Milne's Systems of Particles in General Relativity," Quart. J. Math. 9, 187, Eq. (9) (1938). See also reference 15.

[19] Reference 14, Eq. (6.1).

[20] Reference 14, Eq. (8.8), where we put $\kappa E=12$.

[21] P. A. M. Dirac, "A New Basis for Cosmology," Proc. Roy. Soc. A165, 199–208 (1938), Eq. (6).

[22] Reference 13, p. 824, assumption II′.

It is easily verified that the line elements of the universes E, Mi, M_3, are invariant in form under the transformations (4.81), (4.82), (4.83), respectively, and are therefore stationary. The line element of the universe S_3 is invariant under (4.82), which, however, preserves the origin (0, 0). But this line element is singular at $t=0$, and thus the definition of stationary cosmologies applies to S_3.

The different cosmological forms of the special models examined in this section are summarized in Table IV.

6. THE NEBULAR RED SHIFT

The displacement towards the red of the spectral lines of nebulae, which is roughly proportional to their distance from us, is now a well established astronomical phenomenon.[8] One of the chief advantages of many relativistic models of our universe is that this nebular red shift emerges as a natural consequence of their structure. In this section expressions for the red shift will be obtained in terms of the coordinates of the fundamental particles under observation. No comparison with observational data will be attempted as the coordinates x^i are not immediately interpretable in terms of physical time and distance as estimated by the astronomer. Such a comparison involves an examination of the apparent magnitudes and luminosities of nebulae in the model; the problem is not difficult but is outside the scope of this report. However, in most cosmological models where the red shift is not altogether absent (as it is in the Einstein universe) the first-order red shift effect is linear, and agreement with observation can be achieved by a suitable choice of some of the constants of the model, such as 2α (the cosmological unit) or the present time t_0 (which may, for practical purposes, be treated as a constant). Thus the red shift phenomenon, at least to the first order, is not a very effective criterion for narrowing down the large number of possible cosmological models. Most restrictions on the models, which are suitable for describing our universe at large, are obtained from the red shift effects of higher order and from dynamical considerations which are outside the domain of pure kinematics.

We base the derivation of the red shift formula on a practical, corpuscular theory of light. This is completely justified by the fact that Maxwell's equations have in all cosmological spaces the same form as in Minkowski space and that therefore light is propagated exactly as in flat space. Appeal is also made to a fundamental principle of relativity which states that the proper period of vibration (i.e., the period measured in proper time ds) of an atom emitting a sharp spectral line is constant, whatever the motion or position of the atom. Consider an atom, moving with a fundamental particle $P\equiv(t, r)$ of radial velocity V, emitting a light signal at time t and again, after one complete vibration, at time $t+dt$. The two light signals reach the observer O at the spatial origin at times $t_0=t+r$ and t_0+dt_0, respectively, where $dt_0=dt(1+V)$. This is immediately seen from the fact that the second light signal is emitted from the event $(t+dt,\ r+Vdt)$ and reaches O at time $t+r+dt(1+V)$. Remembering the form of the cosmological line element (3.1), we find that the proper period of vibration of the atom (which equals the proper wave-length λ, since the velocity of light is 1) is given by

$$ds=\lambda=\gamma^{\frac{1}{2}}(t, r)(1-V^2)^{\frac{1}{2}}dt.$$

The proper period of vibration as observed by O is $ds_0=\gamma_0^{\frac{1}{2}}dt_0$, or,

$$ds_0=\lambda_0=\gamma_0^{\frac{1}{2}}(1+V)dt, \quad \gamma_0=\gamma(t_0, 0).$$

Thus the ratio of observed to proper wave-length is

$$\lambda_0/\lambda=(\gamma_0/\gamma)^{\frac{1}{2}}((1+V)/(1-V))^{\frac{1}{2}}. \qquad (6.1)$$

This expression for the red shift may be analysed into two independent components, the Doppler effect which contributes the factor $((1+V)/(1-V))^{\frac{1}{2}}$, and the gravitational effect which contributes the factor $(\gamma_0/\gamma)^{\frac{1}{2}}$.

An explicit formula for the red shift will now be obtained in each of the three types of universes and the result expressed in terms of the time t_0 of observation and the distance r of the fundamental particle P, determined at the time t when the observed radiation left P.

Case I.—The velocity V of a fundamental particle is obtained by differentiating the equation of motion, given in Table III.

$$V=2tr/(t^2+r^2+1). \qquad (6.2)$$

TABLE V. First-order red shift effects.

Universe	Type	γ	$\Delta\lambda/\lambda$ to first order	Remarks
S_1 (a)	I	$(1-a)^{-2}$	$(4t_0/(1+t_0^2))\bar{r}$	$-1<t_0<0$: v.s.* $0<t_0<1$: r.s.
E (c)	I	$4\{(1-a)^2+4t^2\}^{-1}$	0	rigorously
M_2 (d)	II′	1	$(1/t_0)\bar{r}$	$t_0<0$: v.s. $t_0>0$: r.s.
S_2 (h)	II′	$(1-a)^{-2}$	$\frac{\lvert 1-t_0^2\rvert}{1-t_0^2}\cdot\left(\frac{1}{t_0}+t_0\right)\cdot\bar{r}$	$t_0<-1$, $0<t_0<1$: r.s. $-1<t_0<0$, $t_0>1$: v.s.
Mi (j)	II′	$1/a$	0	rigorously
M_3 (l)	III	1	0	rigorously
S_3 (n)	III	$1/4t^2$	$-2(\lvert t_0\rvert/t_0)\bar{r}$	$t_0<0$: r.s.; $t_0>0$: v.s.
ES (q)	III	t^4	$(2/t_0^3)\bar{r}$	$t_0<0$: v.s.; $t_0>0$: r.s.

* r.s. is red shift; v.s. is violet shift.

Replacing t by t_0-r, we have

$$V=2r(t_0-r)/((t_0-r)^2+r^2+1). \quad (6.21)$$

Thus the Doppler effect is given by the expression

$$((1+V)/(1-V))^{\frac{1}{2}} = ((1+t_0^2)/(1+(t_0-2r)^2))^{\frac{1}{2}}. \quad (6.22)$$

Before considering the complete red shift in the general case, it is convenient to examine the special case of the Einstein universe E, whose line element is given by (5.13). Remembering that $\gamma=\gamma(t,r)=\gamma(t_0-r,r)$, we have

$$\left(\frac{\gamma_0}{\gamma}\right)^{\frac{1}{2}}=\left(\frac{(1-(t_0-r)^2-r^2)^2+4(t_0-r)^2}{(1-t_0^2)^2+4t_0^2}\right)^{\frac{1}{2}}, \quad (6.23)$$

which, after simplification, reduces to the reciprocal of (6.22). Thus $\lambda_0/\lambda=1$ in the Einstein universe and there is no red shift.

The Doppler effect is the same for all universes of Type I. Writing $\gamma=F\cdot\gamma_E$, where $\gamma_E\eta_{ij}dx^idx^j$ is the line element of the Einstein universe, we see, from (6.1), and from the fact that $(\gamma_{E0}/\gamma_E)^{\frac{1}{2}}$ just cancels the Doppler effect, that

$$\frac{\lambda_0}{\lambda}=\left(\frac{F_0}{F}\right)^{\frac{1}{2}}=\left(\frac{F(t_0/(1-t_0^2))}{F((t_0-r)/(1-t_0^2+2t_0r))}\right)^{\frac{1}{2}}. \quad (6.3)$$

In terms of the function f, introduced in Section 1, F is defined by

$$F(x)=(\tfrac{1}{4}+x^2)f(x), \quad (6.31)$$

as is easily verified by Eqs. (5.1) and (5.13).

Case II′.—The fundamental particles now move with constant velocity $V=r/t$ and $\gamma=\gamma(a)$. Thus $V=r/(t_0-r)$, and Eq. (6.1) becomes

$$\lambda_0/\lambda=(t_0\gamma(t_0^2)/(t_0-2r)\gamma(t_0^2-2t_0r))^{\frac{1}{2}}. \quad (6.4)$$

It is interesting to note that Milne's universe Mi, for which $\gamma=1/a$, exhibits no red shift.

Case III.—The fundamental particles in these models are at rest and hence the red shift is completely due to the gravitational effect. Remembering that $\gamma=\gamma(t)$, we have

$$\lambda_0/\lambda=(\gamma(t_0)/\gamma(t_0-r))^{\frac{1}{2}}. \quad (6.5)$$

The only universe of this type without red shift is the Minkowski universe M_3.

We proceed to obtain formulas for the first-order red shift effect in the special cosmological models listed in Table IV. The red shift will be given by the usual expression

$$\Delta\lambda/\lambda=(\lambda_0-\lambda)/\lambda. \quad (6.6)$$

It will be expressed in terms of the time t_0 of observation and a new distance coordinate $\bar{r}$, defined by

$$\bar{r}=\lvert\gamma_0^{\frac{1}{2}}\rvert r, \quad \gamma_0=\gamma(t_0,0). \quad (6.61)$$

From the line element (4.3) we note that $\bar{r}$ is the proper distance at time t_0 and in the immediate neighborhood of the spatial origin. Neglecting second-order corrections, $\bar{r}$ is actually the distance estimated by the astronomer, who always assumes the rectilinear propagation of light with constant velocity.

With the formulas (6.3)–(6.5), (6.61), and the functions γ of Table IV, the calculations are quite straightforward. Therefore no computational details need be given and the first-order red shift effects are collected in Table V.

Finally, it is of interest to note that the three models E, Mi, M_3, are the only universes with no red shift.

7. THE PHYSICAL CONTENT OF THE COSMOLOGICAL COORDINATES

Suppose a physical cosmological model be given which is of the kind considered in this report. In this concluding section we deal with the problem of finding the line element of the model and of determining the coordinates of any event in it. This is achieved by means of idealized experiments. They give a clear indication of the physical content of our time and distance coordinates which so far were only convenient mathematical constructs.

We choose a fundamental particle (arbitrarily) and associate with it a coincident fundamental observer O. Without loss of generality, we may assume the particle to be at the spatial origin. The observer O is to be equipped with a theodolite and with apparatus for sending and receiving light signals. Also he is to carry a mechanical or *atomic clock* whose vibrations, according to the fundamental principles of relativity, measure proper time ds.

For the purposes of this section, the most convenient coordinate system is that introduced in Section 4, Eq. (4.2). The line element is

$$ds^2=R^2(\tau)\{d\tau^2-d\rho^2 - S^2(\rho)(d\theta^2+\sin^2\theta d\varphi^2)\}, \quad (7.1)$$

where $S(\rho)$ is $\sin\rho$, $\sinh\rho$, or ρ, according as the model is of type I, II′, or III. From (7.1) simple transformations lead to the form of the line element in Robertson's coordinate system [Eq. (4.11)] or in a c.c.s. [Eqs. (4.4), (4.6), (4.7), in cases I, II′, III, respectively].

Introducing new variables

$$(\bar\tau, \bar\rho)=(\alpha\tau, \alpha\rho), \quad (7.11)$$

where α is a constant, the line element (7.1) becomes

$$ds^2=\phi^2(\bar\tau)\{d\bar\tau^2-d\bar\rho^2 - \alpha^2S^2(\bar\rho/\alpha)(d\theta^2+\sin^2\theta d\varphi^2)\}, \quad (7.2)$$

where $\phi(\bar\tau)=(1/\alpha)R(\bar\tau/\alpha)$. In (7.1), τ, ρ are dimensionless numbers; R has the dimensions of ds. In (7.2), ϕ is dimensionless; $\bar\tau$, $\bar\rho$ have the dimensions of ds. By a suitable choice of α, we can always ensure that

$$\phi(0)=1. \quad (7.21)$$

We know that in the coordinate systems of (7.1) and (7.2) the fundamental particles are at rest. Moreover, the velocity of radial light signals is 1; this is an important property which these coordinate systems share with the c.c.s.

We shall now proceed in the following order: 1. A $\bar\tau$-clock is constructed which measures $\bar\tau$-time at O; it is used to obtain the coordinates $(\bar\tau, \bar\rho)$ of any event. 2. The type of the universe and the constant α are determined. 3. The line element is obtained. 4. The transition to a c.c.s. is examined.

1. The observer O sends a light signal to a fundamental particle P in his immediate neighborhood. P reflects the light ray back to O who returns it to P, and this procedure is repeated indefinitely. The fundamental observer O counts the number of times the light-pulse has reached him and this number serves as a measure of time, $\bar\tau$ time. This is immediately seen from the fact that, in the coordinates of (7.2), all fundamental particles are at rest and that the radial velocity of light is constant.

Since, in the general line element (7.2), there is no preferred point on the temporal axis, O may, without loss of generality, denote the initial moment at which he starts his light ray experiment by $\bar\tau=0$. In accordance with (7.21), O normalizes the $\bar\tau$-time measure to agree initially with the proper time of his atomic clock; i.e.,

$$ds=d\bar\tau, \quad \text{when} \quad \bar\tau=0. \quad (7.22)$$

The mechanism just described will be called a *$\bar\tau$-clock*.

In order to obtain the coordinates $(\bar\tau, \bar\rho, \theta, \varphi)$ of an event E, the observer O sends, at time $\bar\tau_1$, a light signal to E which is reflected by E and returns to O at time $\bar\tau_2$. The direction of the light ray, measured by O's theodolite, immediately yields the polar coordinates θ and φ. Since the light signal travels radially with unit velocity, the coordinates $\bar\tau$, $\bar\rho$ of E are given by

$$\bar\tau=\tfrac{1}{2}(\bar\tau_2+\bar\tau_1), \quad \bar\rho=\tfrac{1}{2}(\bar\tau_2-\bar\tau_1). \quad (7.3)$$

2. The observer O selects two fundamental particles P_1 and P_2 close together and at equal distance from him. By the procedure outlined above, O determines the coordinate distance $OP_1=OP_2=\bar\rho$ and the small angle $P_1OP_2=d\theta$.

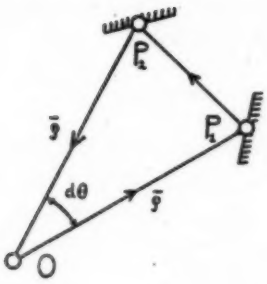

FIG. 4. Arrangement of light signals to determine the type of universe.

O sends a light signal to P_1 who reflects it to P_2, who, in turn, sends it back to O. This arrangement is sketched in Fig. 4. O measures the time interval $\Delta\bar{\tau}$ in which the light ray completes the circuit OP_1P_2O. Then the light ray travels the distance P_1P_2 in time

$$d\bar{\tau}=\Delta\bar{\tau}-2\bar{\rho},$$

and O can compute the transverse velocity of light $\bar{\rho}d\theta/d\bar{\tau}=\bar{c}_t$. From the line element (7.2) it follows that

$$\bar{c}_t=(\bar{\rho}/\alpha)/S(\bar{\rho}/\alpha). \tag{7.4}$$

$S(x)$ is $\sin x$, $\sinh x$, x, in Cases I,II′, III, respectively, and $x/\sin x>1$, $x/\sinh x<1$, $x/x=1$. Thus we have the following criterion:

The given cosmological model is of type I, II′, or III, according as $\bar{c}_t$ is greater than, less than, or equal to 1.

In Cases I and II′, the constant α can now be obtained from Eq. (7.4). In Case III, α does not enter the line element (7.2).

The experiment described above is based on the observation of *transverse effects*. These transverse effects also enter such phenomena as the distribution of nebulae in depth and the second-order red shift effect, when the estimate of nebular distances is based on the observation of apparent luminosities. These phenomena may serve as a practical means of determining the type of the universe and the order of magnitude of the constant α.

3. As described above, O normalizes his $\bar{\tau}$-clock so that, initially, its time measure $d\bar{\tau}$ coincides with the proper-time measure ds of his atomic clock. After a period of time, however, he finds that his two time measures no longer agree. O can, by prolonged observation, determine $ds/d\bar{\tau}$ as a function of $\bar{\tau}$. But, by (7.2),

$$ds/d\bar{\tau}=\phi(\bar{\tau}). \tag{7.5}$$

Thus the function ϕ, and therefore also the line element, are known.

4. Applying the transformation (7.11) to (7.2), we regain the line element (7.1). Regraduating the coordinate time scale in accordance with (7.11), O obtains a *τ-clock*.

From the line element (7.1), the coordinate transformations (4.4), (4.6), or (4.7), according as the cosmological model is of type I, II′, or III, lead to the form of the line element in cosmological coordinates.

In a c.c.s. the observer O can determine the coordinates t, r of an event E by the procedure outlined above for the coordinates $\bar{\tau}$, $\bar{\rho}$. We have

$$t=\tfrac{1}{2}(t_2+t_1), \quad r=\tfrac{1}{2}(t_2-t_1), \tag{7.6}$$

the notation corresponding to that in Eqs. (7.3). However, the structure of O's *t-clock* is different. From the transformation equations it follows, by putting $r=\rho=0$, that the t-clock is characterized by the following equations

$$\text{Case I:} \quad t=\tan\tfrac{1}{2}\tau, \tag{7.61}$$

$$\text{Case II′:} \quad t=e^{\tau}, \tag{7.62}$$

$$\text{Case III:} \quad t=\tau. \tag{7.63}$$

We see that the structure of the t-clock does not depend on the particular function R, as does the τ'-clock measuring Robertson's τ'-time.

This completes the examination which we undertook in this section. However, we now have an opportunity to indicate how the transformations leading to a c.c.s., which we merely stated in Section 4, can be arrived at by logical deduction and without guesswork. In Case III, the line element (7.1) is automatically of the cosmological form, and no transformation is required. We shall limit ourselves to an examination of Case II′, the treatment of Case I being similar.

In the c.c.s. II′, the equation of motion of a fundamental particle P was (in Section 1) shown to be

$$r=vt. \tag{7.7}$$

A light signal is, at time t_1, sent by O to P who reflects it, the light signal returning to O at time t_2. By (7.6) and (7.7), we have

$$(t_2-t_1)/(t_2+t_1)=v. \tag{7.71}$$

We assume that P is close to O, i.e., that v is small. We denote the short cosmological time element t_2-t_1 by dt. The constant $(t_2-t_1)/(t_2+t_1)$, or $dt/2t$, since OP is small, is proportional to a constant τ-interval $d\tau$. Thus

$$d\tau=\beta dt/t, \tag{7.72}$$

where β is a constant. Integrating, we have

$$\tau=\beta \log (t/\beta), \quad t=\beta e^{\tau/\beta}. \tag{7.8}$$

The constant of integration has, without loss of generality, been chosen such that $d\tau=dt$ when $\tau=0$. Regraduating the τ-clock in accordance with Eq. (7.8), we obtain a *t clock* measuring cosmological time.

By use of the t-clock the coordinates of an event E may be obtained from Eq. (7.6). Similarly, the coordinates τ, ρ of E are given by

$$\tau=\tfrac{1}{2}(\tau_2+\tau_1), \quad \rho=\tfrac{1}{2}(\tau_2-\tau_1).$$

Thus

$$\begin{aligned} t_2&=(t+r), \quad \tau_2=(\tau+\rho), \\ t_1&=(t-r), \quad \tau_1=(\tau-\rho). \end{aligned} \tag{7.81}$$

The times t_2, t_1 are connected with τ_2, τ_1, respectively, by (7.8). Thus we obtain the equations of transformation

$$t\pm r=\beta e^{(\tau\pm\rho)/\beta}. \tag{7.9}$$

If this transformation is applied to the line element (7.1), it is seen that the resulting line element is of the cosmological form only if $\beta=1$. Thus the constant β is determined and (7.9) now agrees with (4.6).

Similarly, the transformation leading from the coordinates $(\bar\tau, \bar\rho)$ to cosmological coordinates $(\bar t, \bar r)$, with dimensions, is given by

$$\bar t\pm\bar r=\alpha e^{(\bar\tau\pm\bar\rho)/\alpha}; \quad (\bar t\pm\bar r)/\alpha=t\pm r; \tag{7.91}$$

if we demand that $d\bar t=d\bar\tau$ when $\bar\tau=0$. The constant α which appeared in the line element (7.2) as the negative radius of curvature of the spatial section $\bar\tau=0$, appears now, in the c.c.s., as the age of the universe on the cosmological time scale.

In Case I, parallel considerations show that the transformation equations are

$$\begin{aligned} &(\bar t\pm\bar r)/2\alpha=\tan((\bar\tau\pm\bar\rho)/2\alpha); \\ &(\bar t\pm\bar r)/2\alpha=t\pm r. \end{aligned} \tag{7.92}$$

The radius of curvature α of the spatial section $\bar\tau=0$ appears, in the c.c.s., as one-quarter of the coordinate period of oscillation.

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Note on Magnetic Energy

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1. OBJECT

THE object of this note is to correlate the results obtained in a previous paper[1] concerned with magnetic energy with the contents of a recent paper[2] on this subject by Livens.

I shall show that Livens' formulae are particular examples of my formulae valid for two restricted types of magnetic substances. The assertion by Livens that these are the only types of magnetic substance for which formulae for the energy are obtainable is false, since my formulae apply also to other types.

Livens gives no references to previous treatments of magnetic energy, and I agree with him that most of these are extremely unsatisfactory, but an outstanding exception is that by Cohn[3] already referred to in my earlier paper.

2. NOTATION

The following are the most important symbols used

B	magnetic induction
H	intensity of magnetic field
M	intensity of permanent magnetization
I	intensity of induced magnetization
i	electric current in linear circuit
N	total magnetic flux threading circuit
c	speed of light
$\mathfrak{H}$	Hamiltonian function
$\mathfrak{L}$	Lagrangian function
dV	element of volume

In order to achieve so far as possible uniformity with Livens, I shall follow him in suppressing the permeability of empty space, although I did not do so in my earlier paper, nor for reasons stated elsewhere[4] would I do so as a general practice. If, however, I followed my usual practice, there would result apparent and irrelevant differences between my formulae and those of Livens, and these differences would only obscure the issue. The only outstanding difference between my notation and that of Livens is the symbol for intensity of permanent magnetization; instead of following Cohn I use M rather than I_0. One thus has the universal relation

$$B=H+4\pi I+4\pi M. \tag{2.1}$$

The susceptibility κ and permeability μ are defined by

$$\kappa=I/H, \tag{2.2}$$

$$\mu=1+4\pi\kappa, \tag{2.3}$$

so that

$$B=\mu H+4\pi M. \tag{2.4}$$

Following Livens, I define two other coefficients κ' and μ' by

$$\kappa'=I/B, \tag{2.5}$$

$$\mu'=1-4\pi\kappa', \tag{2.6}$$

so that

$$H=\mu' B-4\pi M. \tag{2.7}$$

3. GENERAL CASE

Consider now a system consisting of linear circuits and magnetic substances. In order to obtain compact formulae valid in the presence of substances with permanent magnetization, it is expedient to postulate that each portion of such substance is surrounded by suitable auxiliary circuits so disposed that it is possible for currents to flow in these circuits such that the magnetic induction due to them is everywhere equal and opposite to that due to the permanent magnetization. When the currents in these auxiliary circuits are adjusted to achieve this object and the currents in all other circuits are zero, the magnetic induction B will vanish everywhere, while H will vanish everywhere except inside the permanent magnets. I shall for the sake of brevity refer to any such state of $B=0$ throughout as a "zero state." The importance of "zero states" is due to the following property: when the system is in a zero state, there are no forces of magnetic origin acting on any piece of magnetic matter complete with its auxiliary circuits, and so each

* Temporarily at Montreal Laboratory, National Research Council of Canada.
[1] E. A. Guggenheim, Proc. Roy. Soc. **A155**, 49 (1936).
[2] G. H. Livens, Phil. Mag. **36**, 1 (1945).
[3] E. Cohn, *Das Elekromagnetische Feld* (1927).
[4] E. A. Guggenheim, Phil. Mag. **33**, 479 (1942).

of these, together with its auxiliary circuits, can be moved relatively to the others without any change of the magnetic energy of the system.

In my previous paper, I showed that if the current i_s in the circuit s is regarded as a generalized velocity, then the corresponding generalized momentum is N_s/c, where N_s is the total magnetic flux threading this circuit, and c is the speed of light. I further showed that for each configuration of circuit and magnetic matter, provided all the N's are uniquely determined by all the i's, the Lagrangian $\mathfrak{L}$ is of the form

$$\mathfrak{L}=\mathfrak{L}_0+\sum_s \int_{N_s=0}^{i_s} \frac{N_s}{c} di_s, \tag{3.1}$$

where $\mathfrak{L}_0$ is the Lagrangian *in the zero state of the same configuration*, and the integration has to be performed *at constant configuration;* $\mathfrak{L}_0$ depends on the configuration and on the velocities, but is independent of the currents and so also of the state of magnetization of the substances in the system. The corresponding formula for the Hamiltonian $\mathfrak{H}$ is

$$\mathfrak{H}=\mathfrak{H}_0+\sum_s \int_0^{N_s} \frac{i_s}{c} dN_s, \tag{3.2}$$

where $\mathfrak{H}_0$ is the Hamiltonian *in the zero state of the same configuration*, and the integration has to be performed *at constant configuration.*

By use of Maxwell's relations it was shown that (3.1) and (3.2) can be transformed to

$$\mathfrak{L}=\mathfrak{L}_0+\frac{1}{4\pi}\int dV \int_{B=0}^{H} BdH, \tag{3.3}$$

$$\mathfrak{H}=\mathfrak{H}_0+\frac{1}{4\pi}\int dV \int_0^{B} HdB, \tag{3.4}$$

where the first integration extends over the whole volume of the system (assumed to have a boundary where $B=H=0$), and the second integration is performed *at constant configuration.* I showed that the only requirement for the validity of formulae (3.3) and (3.4) is that *B and H should be single-valued functions of each other.* In particular it is *not necessary* that they should be *linear* functions of each other. Only hysteresis must be excluded.

For the sake of brevity, I shall henceforth denote by l the contribution per unit of volume to the magnetic term in the Lagrangian and by w the contribution per unit of volume to the magnetic term in the Hamiltonian (or energy). Thus

$$\mathfrak{L}=\mathfrak{L}_0+\int l dV, \tag{3.5}$$

$$l=\frac{1}{4\pi}\int_{B=0}^{H} BdH; \tag{3.6}$$

$$\mathfrak{H}=\mathfrak{H}_0+\int w dV, \tag{3.7}$$

$$w=\frac{1}{4\pi}\int_0^{B} HdB. \tag{3.8}$$

The integrations in (3.5) and (3.7) extend over the whole volume of the system. The integrations in (3.6) and (3.8) are to be performed at constant configuration.

4. "LINEAR" LAWS OF INDUCTION

The essential difference between my treatment and that of Livens is that mine applies to any unique relation between B and H whereas Livens confines himself to what he calls linear laws of induction. He even refers (p. 16) to "the conservation principle, which exists in fact only when the law of induction follows a linear law" and again (bottom of p. 17) to "the assumption of a linear law of induction, and it is only then that conserved energy exists." If my formulae are correct, these statements are patently untrue. Moreover, as I shall point out below, Livens uses the expression "linear law of induction" in two distinct and generally incompatible senses.

According to the definitions of μ, κ and μ', κ' given in Section 2, one has the relations

$$\frac{1}{1-\mu'}-\frac{1}{1-\mu}=\frac{1}{4\pi\kappa'}-\frac{1}{4\pi\kappa}=1+\frac{M}{I}. \tag{4.1}$$

Now I varies with B and H, while M is a constant. It follows that, except in the trivial case $M=0$, the assumption that μ, κ are constant (that is independent of B, H) implies that μ', κ' vary with B, H and *vice versa*.

The two types of substance for which Livens gives formulae are the following:

I. μ, κ independent of B, H; μ', κ' vary with B, H.

II. μ', κ' independent of B, H; μ, κ vary with B, H.

Livens refers to both conditions indiscriminately as a linear law of induction. It is not clear from his text whether he realizes that the two assumptions are incompatible, except in the trivial case $M=0$. I shall now discuss the two cases in turn.

5. CASE OF μ, κ CONSTANT

In the special case of μ, κ independent of B, H the integration in (3.6) is readily performed, and one obtains

$$\begin{aligned} l&=\frac{1}{8\pi}BH+\frac{1}{2}MH+2\pi\frac{M^2}{\mu}, \\ &=\frac{1}{4\pi}BH-\frac{1}{8\pi}\mu H^2+2\pi\frac{M^2}{\mu}. \end{aligned} \tag{5.1}$$

The last term is a trivial constant, apart from which this is equivalent to the formulae obtained by Livens (4th and 2nd formulae displayed on p. 11).

Under the same conditions the integration in (3.8) is readily performed and one obtains

$$\begin{aligned} w&=\frac{1}{8\pi}BH-\frac{1}{2}MH-2\pi\frac{M^2}{\mu} \\ &=\frac{1}{8\pi}\mu H^2-2\pi\frac{M^2}{\mu}, \end{aligned} \tag{5.2}$$

which is equivalent to formula (11.1) of my earlier paper. The last term in (5.2) is a trivial constant, apart from which this is equivalent to Livens formula (top of p. 13) and to formula (11.3) of my earlier paper.

It is noteworthy that in the absence of permanent magnetization ($M=0$), there is no difference between l and w. This is in accordance with expectation, since it is permissible to regard the energy of electric currents as purely kinetic.

6. CASE OF μ', κ' CONSTANT

In the different case of μ', κ' independent of B, H the integration of (3.6) is again readily performed, and one obtains

$$\begin{aligned} l&=\frac{1}{8\pi}BH+\frac{1}{2}MB \\ &=\frac{1}{8\pi}\mu' B^2, \end{aligned} \tag{6.1}$$

again in agreement with Livens (last formula on p. 17).

Under the same conditions integration of (3.6) gives

$$\begin{aligned} w&=\frac{1}{8\pi}BH-\frac{1}{2}MB \\ &=\frac{1}{4\pi}BH-\frac{1}{8\pi}\mu' B^2. \end{aligned} \tag{6.2}$$

which agrees with Livens (last but one formula on p. 12).

Again in the absence of permanent magnetization ($M=0$), there is no difference between l and w.

7. DISCUSSION OF ALTERNATIVE ASSUMPTIONS

In the absence of permanent magnetization (4.1) reduces to

$$\mu\mu'=1, \tag{7.1}$$

and the alternative assumptions constant μ and constant μ' become equivalent. It is immediately evident that when $M=0$, each formula of Section 6 becomes identical with the corresponding formula of Section 5.

When $M\neq 0$ the two assumptions of constant μ and of constant μ' are incompatible. The former is the more usual, but there is no ground for this other than habit. Livens suggests (Sections 12 and 13) that the latter should be a more realistic assumption. Actually I doubt if either assumption should be regarded as anything more than a rough empirical approximation.

Experimentally it is difficult to obtain sufficiently accurate data completely free from hysteresis to distinguish between the two assumptions and, as already mentioned, none of the formulae hold rigorously when there is hysteresis.

According to the only available theory[5] of an idealized ferromagnetic without hysteresis, the

[5] E. C. Stoner, Proc. Roy. Soc. A165, 372 (1938).

relation between B and H must be much more complicated than corresponds to either of the assumptions constant μ or constant μ'.

8. NATURE OF B AND H

The concluding Sections (12, 14 and 15) of Livens' paper are largely devoted to the question which of the two vectors B and H is the "fundamental (aethereal) force vector" and Livens considers that it is B, without however explaining what he means by "fundamental (aethereal) force vector." The following remarks concerning B and H may perhaps be relevant.

Just as the force on an elementary static charge is determined by the electric field intensity E, so the force on an element of electric current is determined by B. Moreover, just as the electric displacement D is closely related by one of Maxwell's equations to the distribution of electric charge, so is H related by another of Maxwell's equations to the distribution of electric current. In this respect it may be said that B is the analog of E, while H is the analog of D. This analogy has been pointed out in many other places and in particular by Sommerfeld,[6] but is unfortunately obscured in many of the best known textbooks. Moreover, in special relativity theory E and B are parts of the same 6-component antisymmetric tensor while D and H are parts of another such tensor. It seems unprofitable to discuss which of these tensors is the more "fundamental" or the more "aethereal."

The fact that the roles of B and H in the Lagrangian and the Hamiltonian are *not* analogous to those of E and D is consistent with the treatment of magnetic energy as kinetic and the electrostatic energy as potential. This is discussed in detail in my earlier paper (p. 63).

[6] A. J. W. Sommerfeld, Zeits. f. tech. Physik **16**, 420 (1935).

Letters to the Editor

PROMPT publication of brief reports of important discoveries in physics may be secured by addressing them to this department. The closing date for this department is the third of the month. Because of the late closing date for the section no proof can be shown to authors. The Board of Editors does not hold itself responsible for the opinions expressed by the correspondents. Communications should not in general exceed 600 words in length.

Decay Schemes for Isotopes W^{187} and W^{185}

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November 13, 1945

INVESTIGATIONS since the earliest reports[1,2] have added considerably to our knowledge of the radiations associated with the 24.1 hour W^{187} activity. By coincidence absorption measurements, Clark[3] has shown that the beta-radiations are complex, with maximum energies of 0.5 Mev and (roughly) 1.3±0.1 Mev. A careful analysis of our previously reported beta-ray absorption curves, using the Feather comparison method,[4] has confirmed his observations. The ranges of 600 mg/cm² Al and ~140 mg/cm² Al correspond to energies of 1.4 Mev and 0.5$_8$ Mev, respectively, according to the Feather relation.[4]

Clark observed also that beta-gamma coincidences occurred only with aluminum absorbers thin enough to permit passage of the 0.5 Mev negatrons, showing that the 0.9 Mev gamma-ray was associated with the softer beta-radiation. At the same time gamma-gamma coincidences were found, indicating that other gamma-rays occurred in parallel with the 0.9 Mev gamma-transition. From measurements giving the number of gamma-gamma coincidences per recorded gamma ray and the number of beta-gamma coincidences per recorded beta-ray, Clark calculated that four low energy gamma-rays were emitted simultaneously, with a branching ratio of 0.4. Thus, he proposed the decay scheme shown in Fig. 1.

In other studies, Valley[5] has examined the complicated spectrum of internal conversion electrons emitted by tungsten specimens irradiated with 11 Mev deuterons and has found that six of the lines, decaying with a 24-hour half-life, are associated with gamma-rays having energies of 0.086, 0.101, and 0.135 Mev. More recently, Mandeville[6] has redetermined the energy of the hard gamma-radiation

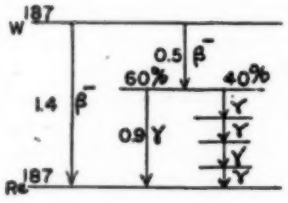

FIG. 1. Decay scheme for the 24.1-hour W^{187} as proposed by Clark. Energy changes are in Mev.

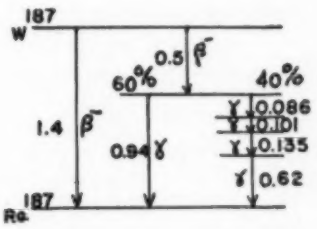

FIG. 2. A proposed decay scheme for the 24.1-hour W^{187} isotope. The 0.086, 0.101, and 0.135 Mev transitions are largely converted.

by measurements of the Compton recoil electrons in a magnetic ray spectrograph and has obtained a value of 0.94±0.02 Mev.

The correlation of the above data into a unified picture for the decay scheme may be initiated by assuming that three of the four soft gamma-rays postulated by Clark from coincidence measurements were observed by Valley. Accordingly, a fourth gamma-ray, with an energy of 0.94 −(0.086+0.101+0.135)=0.62 Mev should be present.

Evidence suggesting the existence of this gamma-ray may be obtained from absorption data in lead for the W^{187} gamma-rays. The absorption curves appear to have a half-value thickness of 7.0 g/cm² Pb. By use of the absorption coefficient *versus* energy curves derived from the data of Heitler[7] and Gentner,[8] which have been found applicable in other measurements with the same experimental set-up, the energy of the gamma-ray would appear to be 0.7 Mev. Since Mandeville[6] has shown that the hard gamma-ray has an energy of 0.94±0.02 Mev, one must conclude that the absorption curve in lead is composed of two components. A rough graphical analysis of this curve, assuming that the hard gamma-ray is about 1.5 times more intense than the soft component, gives a half-value thickness of ~5 g/cm² Pb for the lower energy radiation. This value corresponds to an energy of ~0.5$_8$ Mev, which is in reasonable agreement with the predicted value of 0.62 Mev, considering the errors of such an analysis. Thus, if these deductions are valid, the probable decay scheme for isotope W^{187} is that shown in Fig. 2.

In previous investigations of the 75-day W^{185} activity, maximum beta-ray energies of 0.4–0.5 Mev[1] and 0.55–0.65 Mev[2] have been reported. Additional aluminum absorption measurements with stronger sources have confirmed the 0.55 Mev value for the energy, by use of the Feather comparison method, and have indicated also that no gamma-rays accompany the decay of this isotope, as deduced from absorption curves in aluminum which show beta/gamma ratios of ≳3000 (at zero absorber). Since cloud-chamber measurements have indicated a normal spectrum, it would appear that the decay scheme is simple, according to the mechanism W^{185} (β^-/0.55 Mev) Re^{185}.

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[1] O. Minakawa, Phys. Rev. 57, 1189 (1940).
[2] K. Fajans and W. H. Sullivan, Phys. Rev. 58, 276 (1940).
[3] A. F. Clark, Phys. Rev. 61, 242 (1942).
[4] N. Feather, Proc. Camb. Phil. Soc. 34, 599 (1938).
[5] G. E. Valley, Phys. Rev. 59, 686 (1941).
[6] C. E. Mandeville, Phys. Rev. 64, 147 (1943).
[7] W. Heitler, *The Quantum Theory of Radiation* (The Oxford Press, 1936).
[8] W. Gentner, Physik. Zeits. 38, 836 (1937).

Positronium

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November 13, 1945

1. IN 1937 I conceived the idea that an unstable atom composed of a positron and a negative electron may exist in quantities sufficient for spectroscopic detection. The name *positronium* is suggested. The spectrum of positronium would have lines at wave-lengths twice as great as the hydrogen lines. The first line of the Lyman series would lie at 2430A and the Balmer series would run as follows: 13126A, 9722A, 8680A, and so on, to the limit at 7290A. It is possible that cosmic-ray positrons passing through great thicknesses of nebulous matter might excite this spectrum. A brief survey showed the paucity of our knowledge of the infra-red spectra of planetary nebulae. No indication of the lines was obtained. It was keenly realized that the great probability for annihilation of the atom would reduce the probability of detecting it in this way. The life of a positron in lead is about 10^{-10} sec. The life of the lower levels of free positronium should be greater. Thus the probability of a fall from one discrete energy state to another may be about 0.05 to 0.1 of the annihilation probability. For these reasons nothing was published. Now the situation has altered. Strong beams of positrons are available to workers with the betatron and this note is intended to call the subject to their attention.

2. Some interesting properties of positronium will be listed. Neglecting the effect of annihilation for the instant, we note that the Bohr radius is twice that for hydrogen. The approximate wave functions will be those of hydrogen, with the Bohr radius doubled. Finer details can be treated by the theories of Breit or of Möller. Some of the results of a complete calculation can be foreseen. In discussing hydrogen, the magnetic moment of the proton is neglected because its influence on the spectrum is so small. In discussing the fine structure of positronium, the spins of both particles must be considered; there will be both singlet and triplet states. Also a correction due to Darwin[1] must be included. However, all the states will be broadened by the large annihilation probability. It appears likely that this effect will obscure the fine structure. None of the discrete states are excluded by Pauli's principle.

3. The type of experiment required in a search for the line spectrum of this material consists in passing an intense beam of positrons into a gas of low atomic number, preferably helium. To increase the spectral intensity, high pressure should be employed. If the absolute intensity can be made sufficient, the question remains whether continuous spectra from the gas atoms will be more intense than the radiations sought for. The radiation of atoms which pick up a slow positron must also be considered. In another type of experiment, the modified spectrum produced by a liquid or solid could be examined; Cerenkov radiation would introduce a complication.

I think no physicist will doubt the existence of these unstable hydrogen-like atoms. Direct detection may prove difficult, but the great interest of such observations requires no emphasis.

4. Since writing the above, Professor John A. Wheeler has told me that he has submitted a paper for publication by the New York Academy of Sciences, entitled "Electromesons; Short Lived Entities Composed of Electrons and Positrons," which deals with questions like those discussed in this note. It has been a pleasure to learn of his ideas, which cover possibilities much broader than the simple case of a single electron and a single positron.

* On leave from the University of North Carolina, Chapel Hill, North Carolina.
** Published by permission of the Secretary of the Navy. The views expressed are not to be interpreted as the official views of the Navy.
[1] C. G. Darwin, Phil. Mag. 39, 537 (1920); see brief account in Ruark and Urey, *Atoms, Molecules and Quanta*, p. 167.

Calibration of Ionization Gauge for Different Gases

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November 23, 1945

DURING the past few months we have attempted to calibrate ionization gages for a number of different gases. Table I shows calibration data for two FP-62 tubes and a VG-1 tube. In all cases the measurements were made with i_e (electron current) varying from 0.5 to 5 ma, and collector potential of -22 volt. The anode potential was 125 volts in the case of the FP tubes and 150 volts in that of the VG-1.

Denoting the pressure in microns by P, and the positive ion and electron currents by i_p and i_e, respectively, $s = i_p/(i_e P)$ and r denotes the ratio of s for the gas to that for argon. As will be observed the values of r for different gases appear to be independent to a large extent of the type

TABLE I. Calibration of ionization gauges with argon as a reference gas.

Gas	FP-62(1) $s\times10^3$	FP-62(1) r	FP-62(2) $s\times10^3$	FP-62(2) r	VG-1 $s\times10^{-3}$	VG-1 σ	Av. value r	$r(P_i)$
He	0.77	0.140	0.70	0.133	2.88	0.127	0.133	0.14
Ne	1.08	0.196	1.08	0.205	4.65	0.206	0.202	0.21
A	5.50	1.0	5.27	1.0	22.6	1.0	1.0	1.00
Kr	8.9	1.62	8.4	1.59	32.9	1.46	1.56	—
Xe	12.0	2.16	13.0	2.46	50.9	2.25	2.29	—
Hg	15.0	2.73	14.5	2.75	72.0	3.18	2.89	1.90
H_2	2.14	0.39	2.10	0.40	8.65	0.38	0.39	0.34
N_2	4.90	0.89	4.40	0.83	18.2	0.81	0.84	0.90

of gauge used. The average values of r are in reasonably good agreement with relative values of the ionization probabilities at 100 volts as obtained by I. Langmuir and H. A. Jones.[1] These are shown in the last column of the table under $r(P_i)$.

It is interesting to note that in the case of the rare gases a plot of log r *versus* the ionization potentials deviates only slightly from a straight line.

A more detailed account of the measurements will be published in the near future.

[1] I. Langmuir and H. A. Jones, Phys. Rev. 31, 357 (1928).

Production of Photons in a Townsend Gap in Air, Nitrogen, and Argon

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November 13, 1945

RECENTLY, Geballe[1] reported measurements in a Townsend gap of the photon-ion ratio in hydrogen. This study has been extended, with the same equipment, to air, nitrogen, and argon. No photoelectric currents could be measured below breakdown in any of the three gases studied, regardless of values of pressure and voltage. The electrometer used was capable of detecting 10^{-15} ampere.

In order to detect photoelectrons at the brass surface, photons of adequate energy must be produced in the gas, and these high energy photons must not be absorbed too strongly in the gas. High energy photons may be expected especially in gases having metastable states, and photon absorption will be high in mixed gases and in gases having metastable states. Streamers (which depend on photo-ionization in the gas) may be expected in mixed gases, and in gases containing metastables. Weissler[2] has shown that no pre-onset streamers appear in the positive point-to-plane discharge in pure hydrogen and in pure nitrogen. In argon, the first streamer is so intense that the gap breaks down immediately.

In pure hydrogen, high energy photons reach the photoelectric surface. Therefore streamers are not formed because of the transparency of the gas to photons. Photoelectrons in the gas are produced too far from the positive ion space charge to be effective in producing streamers. In air, high energy photons are produced very close to the space charge, hence one observes streamers but no photoelectric action at the surface. In argon, as in air, the photon absorption is so high that one observes no photoelectric effect at the surface. In nitrogen, no streamers are formed, and no photoelectric currents were measured. This can be understood if one assumes that photons in nitrogen are not very energetic.

[1] R. Geballe, Phys. Rev. 66, 316 (1944).
[2] G. L. Weissler, Phys. Rev. 63, 96 (1943).

Evidence of Increased Radioactivity of the Atmosphere after the Atomic Bomb Test in New Mexico

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November 23, 1945

FROM July 12 through July 19, 1945 a G-M counter and circuit similar to that previously described in *The Review of Scientific Instruments*[1] was being tested for background count in my home near Annapolis. The following data were obtained (Table I). Observations on August 16 and from October 31 through November 3 confirm the normal background of 5.5 counts per minute.

Rain fell from the 15th through the 18th of July, the weather clearing during the latter part of the counting period on the 18th and remaining clear through the counting period on the 19th. No change of background count had been observed during rainy periods previous to the above dates. During the past three weeks a series of more than twenty observations under different conditions of temperature and humidity produced by weather changes have been made. The background count has been normal during these recent observations.

TABLE I.

Day	Time of count	Counts per minute	Hours after bomb test to middle of counting period
12	1500 to 2200	5.5	
14	1600 to 2400	5.5	
15	1500 to 1900	5.5	
16	1900 to 2300	6.3	12.5
17	2000 to 2300	7.7	37.5
18	1600 to 2300	10.9	59
19	1300 to 1500	5.5	78

* Lieutenant Commander, USNR, on leave from Kent State University, Kent, Ohio. The assertions herein are the private ones of the writer, and are not to be construed as official or reflecting the views of the Navy Department or of the Naval Service at large.
[1] A. W. Coven, Rev. Sci. Inst. 13, 188 (1942).

Nuclear Spectroscopy and Inelastic Scattering of Particles by Nuclei

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September 12, 1945

THE purpose of this and of the following note is to discuss the excitation[1] of stable nuclei by inelastic scattering of particles and, in particular, to discuss the detection of this excitation by study of the energy distribution of inelastically scattered deuterons and electrons.

Excitation of nuclei by inelastic scattering may be detected through: (*a*) γ-rays from the excited states, (*b*) decay of a metastable state with which the excited states intercombine (the lowest of these excited states is the threshold for this process), (*c*) energy distribution of inelastically scattered particles, and (*d*) recoil of the nucleus.

Method (*a*) may be applied by using protons, deuterons, alphas, and fast neutrons. Detection of γ-rays is hardly possible in the presence of strong x-rays accompanying an electron beam. Method (*b*) applies only to nuclei possessing a metastable state only; in addition, it yields only a certain part of the nuclear spectrum. Method (*c*) seems to be the most universal method. Method (*d*) may be applied by using p, d, α, n, but it will be quite laborious; apparently, it has not as yet been used.

(*a*) *Fast neutrons* striking medium and heavy nuclei yielded γ-rays according to Kikuchi, *et al.*[2] No quantitative study of the γ-ray energies was made, however. *Protons* were used by Herb, *et al.*[3] (Li^6, Li^7). Schnetzler[4] and Savel[5] used alphas to excite Li^6. The excited level found at about 0.6 Mev by Schnetzler[4] and that at 0.45 Mev found by Herb, *et al.*[3] may be identical; further study is needed.

(*b*) *Fast neutrons* excite In^{115} as discovered by Goldhaber, Hill, and Szilard.[6] For some elements the γ-rays observed by Kikuchi, *et al.*[2] may have originated by this process. *Protons* were used by Barnes and Aradine[7] and *alphas* by Lark-Horovitz, Risser, and Smith[8] to excite In^{115}. *Electrons* were used by the Notre Dame group[9] to

excite In^{115} and Cd. X-rays were employed more extensively by the Notre Dame group[10] to excite a whole series of nuclei. The theoretical basis for this method was given by the author.[11]

(*c*) The general idea of this method is that charged particles are scattered elastically (Rutherford) mainly in the forward direction; *inelastically* scattered particles will tend to a spherically symmetrical angular distribution, however. This difference may enable the detection of *inelastically* scattered (through 90° and backward) particles, although for medium and heavy nuclei the cross section for inelastic scattering may be by orders of magnitude smaller than that of the elastic scattering. *Protons* scattered by a few *light* nuclei were used by Wilkins and Kuerti[12] (Mg, Al) and by Powell, May, Chadwick, and Pickavance[13] (Ne^{20}, Al) employing a photographic method. A counter method was employed by R. H. Dicke and J. Marshall, Jr.[13] (Al^{27}, Cr^{52}, Mg, S^{32}). The protons penetrate the Coulomb barrier and are captured by the nuclei. The resulting compound nucleus re-emits the protons with spherically symmetrical angular distribution. Their number is (for such light nuclei) of the same order (about $\frac{1}{2}$) as that of the elastically scattered protons. Powell, *et al.*[14] point out that the probability of this (p,p) process decreases rapidly with increasing nuclear charge. The same holds for excitation by (d,d) and—*a fortiori*—by an (α,α) nuclear reaction. Cl and A gave too few inelastically scattered particles to observe.

[1] M. S. Livingston and H. A. Bethe summarized the evidence up to 1937. Rev. Mod. Phys. 9, 245 (1937). Recently, P. Comparat (cf. F. C. Champion, Nature 153, 720 (1944)) obtained a large number of levels of N^{15} employing the interesting though laborious method of recoil.
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[3] C. M. Hudson, R. G. Herb, and G. J. Plain, Phys. Rev. 57, 587 (1940).
[4] P. Schnetzler, Zeits. f. Physik 95, 302 (1935).
[5] X. Savel, Comptes rendus 198 (1934).
[6] M. Goldhaber, R. D. Hill, and L. Szilard, Phys. Rev. 55, 47 (1939).
[7] S. W. Barnes and P. W. Aradine, Phys. Rev. 55, 50 (1939).
[8] K. Lark-Horovitz, J. R. Risser, and R. N. Smith, Phys. Rev. 55, 878 (1939).
[9] B. Waldman and M. L. Wiedenbeck, Phys. Rev. 63, 60 (1943); M. L. Wiedenbeck, Phys. Rev. 67, 92 (1945). The work at Notre Dame was initiated by G. B. Collins and B. Waldman and further developed by M. L. Wiedenbeck.
[10] M. L. Wiedenbeck, Phys. Rev. 68, 1 (1945); M. L. Wiedenbeck, Phys. Rev. 67, 267 (1945).
[11] E. Guth, Phys. Rev. 59, 325 (1941).
[12] T. R. Wilkins and G. Kuerti, Phys. Rev. 57, 1082; 58, 758 (1940).
[13] R. H. Dicke and J. Marshall, Jr., Phys. Rev. 63, 86 (1943).
[14] C. G. Powell, A. N. May, J. Chadwick, and T. G. Pickavance, Nature 145, 893 (1940).

Nuclear Spectroscopy and Energy Distribution of Charged Particles Inelastically Scattered by Nuclei

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September 12, 1945

IN the foregoing note various methods of detecting nuclear excitation caused by inelastic scattering of charged particles were discussed. Method (*c*), *viz.* measurement of the energy distribution of the inelastically scattered particles will now be discussed for the cases in which the scattered particles are (1) protons or alphas, (2) deuterons, and (3) electrons.

1. *Protons or alphas.* Besides the (p,p), (α,α) and (d,d) nuclear reactions, excitation of a nucleus may be brought about by the Coulomb field of the approaching charged particle. In the excitation of In^{115} by protons this process may play some role, and in the excitation of this nucleus by alphas it may make the main contribution. The cross section for the excitation of a nucleus of charge Ze by the Coulomb field of a charge ze has the form,

$$\sigma = f(v) \exp\left[\frac{-2\pi e^2 zZ'}{\hbar}\left(\frac{1}{v'}-\frac{1}{v}\right)\right],$$

where v and v' are the velocities of the particle before and after the collision. The form of $f(v)$ depends upon the multipole-character of the transition involved, while the exponential factor (which varies much faster than $f(v)$), probably does not.[1] Because of this exponential factor, transfer of energy from a charged particle to the nucleus can happen with appreciable probability only if $v \gg v'$. Even then methods (e.g., using counters) more sensitive than the photographic method will be necessary for the detection of inelastically scattered particles.

2. *Deuterons.* The use of deuterons yields a somewhat different picture. Because of its low binding energy the deuteron may be polarized by the nuclear field. In this way the neutron may come close enough to the nucleus to enable an energy transfer from the deuteron to the nucleus without the necessity of a (d,d) reaction or even excitation by the Coulomb field of the deuteron. The new process ("polarization scattering") is somewhat similar to the well-known Oppenheimer-Phillips process, but differs from it essentially in that the outgoing particle is not a proton but a deuteron. This difference makes it possible to distinguish between the two processes,[2] even when the conditions for the O-P process are fulfilled (kinetic energy of the deuteron larger than its binding energy, medium, and heavy nuclei). In^{115} for instance should be excitable by deuteron with an energy less than its binding energy. The threshold for the process is just slightly higher than that for excitation by x-rays, because of the recoil of the nucleus.[3] Observation of the inelastically scattered deuterons seems to be a promising method for studying nuclear spectroscopy. Polarization scattering may be used, among other things, to liberate neutrons from Be^9 and D, etc.

3. *Electrons.* Inelastically scattered electrons will not be easy to detect, even in the case in which the scattering angle approaches 180°, because of the background of the elastic scattering. Still a suitable β-ray spectrograph employing sensitive counters should make detection possible. Once established, the use of electrons for nuclear spectroscopy may very well be preferable even to the use of deuterons. For in contrast to the case with deuterons only electrons with comparatively high energies (namely, energies greater than the threshold for photo-disintegration) can disintegrate nuclei.

[1] A rigorous derivation of this formula (including the exact form of $f(v)$) was given by V. F. Weisskopf and the author (unpublished). An earlier derivation of the exponential factor by L. Landau, Physik. Zeits. U.S.S.R. 1, 88 (1932) is not very clear and does not give the value of $f(v)$. The author is indebted to Dr. S. N. Dancoff for pointing out that in the formula giving the cross section for nuclear excitation, followed by re-emission, the same exponential factor enters while the factor $f(v)$ is replaced by another $g(v)$ which in general is much smaller than $f(v)$.
[2] The same holds for excitation of the nucleus by the deuterons disintegrated by the Coulomb field of the nucleus, and for the ordinary (d,d) reaction, mentioned before.
[3] This process has actually been observed for In^{115} by M. L. Wiedenbeck (private communication).

Proceedings of the American Physical Society

Minutes of the Meeting of the Metropolitan Section held at Columbia University, New York, November 9 and 10, 1945

A MEETING of the Metropolitan Section of the American Physical Society, the first of the 1945–1946 season, was held on Friday and Saturday, November 9th and 10th, 1945, in the Pupin Physics Laboratories at Columbia University. In view of the paucity, in recent years, of meetings at which physicists could gather to discuss their research, and since the termination of the European and Japanese wars had relaxed the military and war-connected duties of many physicists, it was decided to prepare a program on a scale sufficiently large to ease the log-jam of accumulated research findings of those in the Metropolitan area.

During the two-day session there were presented four invited lectures and thirty contributed papers, as set forth in the programme below. In addition, the Section held a business session, not originally scheduled, for the sole purpose of considering a set of resolutions which had previously been submitted by an informal gathering of physicists. These resolutions were, in essence, a plea for freedom in the dissemination of fundamental scientific knowledge and a suitable international control of weapons of war. After three hours of spirited discussion, the Section formally adopted the resolutions in a modified form. They will be suitably publicized elsewhere.

The attendance at the scientific sessions varied from 150 to 250 persons. A dinner, held Friday evening at the Mens' Faculty Club, was attended by 123 persons.

William H. Crew
Secretary, Metropolitan Section

Abstracts of Contributed Papers

1. On the Nature of the Forces Involved in the Reaction between Antigen and Antibody Molecules. Alexandre Rothen, *The Laboratories of The Rockefeller Institute for Medical Research, New York.*—Specific fixation of homologous antibodies by antigen molecules occurs when the antigen molecules are deposited on a metal slide, in a completely unfolded film 6 to 8A thick. The specific fixation has been demonstrated by an increase in thickness in the adsorbed layer on the plate. Thicknesses are measured optically with an accuracy of ±0.3A. In the system film of bovine albumin-antibovine albumin rabbit serum, the amount of fixed antibodies increases with the number of underlying films of antigen. Screens consisting of films of stearic acid, octadecylamine or protein deposited on top of the antigen film do not prevent the reaction from occurring to a considerable extent. In certain cases 8 layers of stearic acid are needed to prevent any fixation. Facts are given which speak against the presence of holes in the screens through which the antibodies could reach the antigen. The assumption is tentatively made that the effective range of action between a film of antigen and homologous antibodies might extend to an order of hundreds of A. It is considered possible that such a long range might result from an integrated action of the many elementary units which build up the single large molecules of antigen and antibody in an orderly way.

2. The Sensitivity of the Human Eye on an Absolute Scale. Albert Rose, *RCA Laboratories, Princeton, New Jersey.*—When the sensitivity of a picture reproducing device (e.g. television pick-up tube or photographic film) is limited by the statistical fluctuations inherent in the primary photo-process,[1] the limitation may well be taken to be absolute in the sense that no subsequent link in the chain that forms a picture can avoid it. The performance of the human eye in the range of 10^{-6} to 10^{2} footlamberts is compared with the performance of a model for which the quantum efficiency and statistical fluctuations of the primary photo-process constitute the significant limitations. The performance of the model is given by $B^{\frac{1}{2}}\alpha C = K$ where B is scene luminance, α is minimum resolvable angle, C is contrast, and K is a parameter. The agreement is sufficiently good to recommend the model at least as a summary device. If, in addition, it is taken to be a valid representation, reasonable explanations are suggested for (a) the large ratio between the "sensitivities" of the dark and light adapted eye; (b) the experimental fact that scenes whose luminosities are widely different may still be equally resolved by the eye; (c) the discrepancy between the computed[2] and observed[3] numbers of quanta required for threshold vision. These are one and five, respectively; (d) the observation of fluctuations at very low lights.

[1] A. Rose, Proc. I.R.E. **30**, 215–300 (1942).
[2] H. DeVries, Physica **10**, 553–564 (1943).
[3] S. Hecht, J. Opt. Soc. Am. **32**, 42–49 (1942).

3. Electrodynamic Theory of Piezoelectric Oscillations.* W. F. G. SWANN, *Bartol Research Foundation of the Franklin Institute, Swarthmore, Pennsylvania.*—The problem concerned is that of an x-cut crystal with self-induction and resistance in series, and vibrating with its two ends in different media. It is solved on the basis of Maxwell's general dynamical theory. If p is 2π times the applied frequency, if k is 2π times the lowest natural frequency, and n is an integer, it turns out that if p satisfies the relation, $p^2=n_2^2k^2$, where n_2 is even, there is no resonance, but the crystal vibrates in an infinite number of "odd" modes. If $p^2=n_1^2k^2$ where n_1 is odd, the whole displacement at the two ends of the crystal is given by one single harmonic term of frequency determined by n_1, and the displacements are the same at the two ends. In the latter case, and where the media are the same at the two ends, the radiation pressures are entirely provided by the piezoelectric force which is symmetrical in its action for this purpose. However, when the media are different there arise an infinite number of "even" modes, which add up to zero displacement at the two ends but give unsymmetrical pressures at these ends. Complete expressions for the relevant amplitudes and phase angles are determined.

* Carried on by funds furnished by OSRD.

4. The Order of Magnitude of Piezoelectric Effects. HANS JAFFE, *The Brush Development Company.*—Practical applications of piezoelectric effects until recently were limited to quartz which has a rather low effect and Rochelle salt whose use at elevated temperatures or high power levels is restricted by its decomposition point at 56°C. The selection of piezoelectric materials for various applications depends on the values of different piezoelectric coefficients. Motor devices in air (sound generators) depend on the piezoelectric moduli d. The exceptional value of d_{14} of Rochelle salt justifies its dominant position for such purposes. Voltage generating devices (pick-ups and microphones) operating into typical high impedance circuits require a high "voltage output coefficient" $g=d/\text{diel. const.}$, as well as a high modulus d. Square root of $g\cdot d$ is a suitable figure of merit. In this figure Rochelle salt X-cut is still leading but is ahead of certain more stable crystals only by a factor 3. For generating and receiving ultrasonic waves in liquids, a fairly low elastic compliance, s, is preferable. We arrive at a figure of merit root of $g\cdot d/s$; this is substantially the coupling coefficient which is also decisive for application in filters and oscillator control. Coupling coefficients up to 30 percent are not uncommon among synthetic crystals. Maximum values for the "voltage output coefficient" g can be estimated from atomic models.

5. Relation Between Darkening by X-Ray Irradiation and Permanence of Dauphiné Twinning in Quartz. ELIZABETH ARMSTRONG, *Bell Telephone Laboratories.*—There is a positive correlation between the amount of darkening caused in quartz plates by x-ray irradiation and the permanence of their Dauphiné ("electrical") twin boundaries when subjected to inversion (at 573°±1°C) to the high temperature form and reinversion to low quartz. Of forty-five quartz plates so treated 33 percent resumed their former orientation in all parts of the plate, 11 percent showed changes in less than 5 percent of the plate and 56 percent showed changed orientation in more than 5 percent of the plate. Each plate was then irradiated with x-rays from a copper-target tube at 60 kv and 25 ma for 2 hours. Of the unchanged plates 73 percent became dark, the rest were only slightly darkened. Of the slightly changed plates 80 percent became dark. Of the changed plates 16 percent became dark, 48 percent were only slightly darkened, and 36 percent were inhomogeneously colored. In 75 percent of these last, the lighter areas contained the changed parts of the crystal plate. This correlation between irradiation-darkening and orientation-permanence indicates a common cause for both which is most probably the presence of impurities or defect structures or both.

6. The Elastic, Piezoelectric and Dielectric Constants of Potassium Dihydrogen Phosphate (KDP) and Ammonium Dihydrogen Phosphate (ADP). W. P. MASON, *Bell Telephone Laboratories.*—Measurements have been made of all the elastic, piezoelectric, and dielectric constants of KDP and ADP crystals through temperature ranges down to the Curie temperatures. The piezoelectric properties agree well with Mueller's phenomenological theory of piezoelectricity provided the fundamental piezoelectric constant is taken as the ratio of the piezoelectric stress to that part of the polarization due to the hydrogen bonds. It is found that the dielectric properties of KDP agree well with the theory presented by Slater based on the interaction of the hydrogen bonds with the PO_4 ions. ADP undergoes a transition at −125°C which results in fracturing the crystal. This transition cannot be connected with the H_2PO_4 hydrogen bond system which controls the dielectric and piezoelectric properties for these lie on smooth curves that do not change slope as the transition temperature is approached. It is suggested that two separate and independent hydrogen bond systems are involved in ADP. The transition temperature and specific heat anomaly appear to be connected with hydrogen bonds between the nitrogens and the oxygens of the PO_4 ions, while the dielectric and piezoelectric properties are controlled by the H_2PO_4 hydrogen bonds.

7. Methods of Orienting and Cutting Synthetic Crystals. W. L. BOND, *Bell Telephone Laboratories.*—Improved methods of orienting, cutting and surfacing synthetic crystals have been developed. These include optically orienting on a mounting board, securing by means of fast-setting cement, grinding a reference face at a predetermined angle from the board edges, sawing with solution cooled abrasive blades and grinding to dimension with abrasive belts. The application of these to ADP is discussed. These methods are also applicable to many other crystals. The optical orientation should use reflections from faces habitually quite perfect and in some cases must be supplemented by x-ray orientation to achieve the highest accuracy of orientation. For crystals that crack easily due to cleavage liquid cooled grinding is used. Sawing with abrasive blades works nicely with the proper choice of coolant.

8. Apparatus for Growing Single Crystals from Solution. A. N. HOLDEN, *Bell Telephone Laboratories.*—In growing large crystals from seeds planted in supersaturated solutions two major problems are (1) to provide for the uniform replacement of saturated by supersaturated solution at the crystal surfaces and (2) to prevent the growth of extraneous seeds. In the apparatus described, these problems are largely solved for materials whose solubilities are increasing functions of temperature. Crystals are grown from seeds mounted on radial arms attached to a central shaft alternately rotated several revolutions in one sense, then in the other. The solution through which they move is carried in a cylindrical container heated at the bottom, and thermally controlled for intermittent or continuous decrease of temperature. Spontaneously formed seeds are carried by the centripetal action of the rotating solution to the center of the bottom of the container, where they are dissolved at the slightly higher temperature of the heated portion. The apparatus has been used for many materials; as applied to ammonium dihydrogen phosphate, it has permitted linear growth rates of about 1/16 inch per day for several months of continuous growth.

9. Ionic Conduction in Ammonium Dihydrogen Phosphate Single Crystals. E. J. MURPHY, *Bell Telephone Laboratories.*—The ionic conductivity of a crystal is given by $\sigma = \Sigma \sigma_{0i} \exp(W_i/RT)$. In the present work the activation energies W_i, and the other conduction constants σ_{0i}, were determined for four $NH_4H_2PO_4$ crystals; namely, a very pure crystal and three others containing, respectively, known small amounts of Ba, Sn, and SO_4. The results show that this crystal is able to conduct with at least three essentially different activation energies; namely, 10.1 (10.9), 15.8 and 20.4 kcalorie/mole. The 20.4 kcalorie conduction is self-conduction, the 10.1 term Ba-conduction, the 10.9 term SO_4-conduction, and the 15.8 term Sn-conduction. Thus, in a given crystal some impurity ions can conduct with essentially equal activation energies, while others require a substantially different activation energy; a spectrum of energies is revealed by the impurities. This is a hydrogen bonded material and the activation energies may depend upon the breaking of H-bonds. The observed values of σ_{0i} were compared with theory by calculating the lattice constant from the conduction data on the basis of simple assumptions. The values obtained are 6.4 Angstroms from Ba-conduction, 4.5A from SO_4-conduction, 9.5A from self-conduction and 182A from Sn-conduction. The lattice constant for the *c*-direction in this crystal is about 7A.

10. Focusing Aberration in the Flat Powder Specimen X-Ray Spectrometer Technique. JOHN P. NIELSEN AND WILLIAM PARRISH, *Philips Laboratories, Inc. Irvington, N. Y.*—In an x-ray spectrometer a flat specimen is far more convenient than a curved specimen where the curvature must be changed for each Bragg angle setting to obtain true focusing. However, the flat specimen gives an aberration which broadens the diffracted "line" on the lower Bragg angle side, and hence it is generally thought that a shift in the line position occurs which gives low Bragg angle measurements. A detailed ray diagram was constructed for a flat specimen for a diverging beam at an arbitrary angle. An analysis of the diagram reveals the nature of the broadening which provides means for evaluating the data obtained by the flat specimen technique. Experimental investigation of this theory was carried out with a new type Geiger counter tube focusing x-ray spectrometer on $\{10\bar{1}1\}$ of low temperature quartz powder.

INVITED PAPERS

Meson-Like Entities Composed of Electrons and Positrons. J. A. WHEELER, *Princeton University.*

I. **The Supersonic Reflectoscope.** F. A. FIRESTONE, *University of Michigan.*

II. **Present Aspects of the Theory of Color Centers in the Alkali-Halides.** F. SEITZ, *Carnegie Institute of Technology.*

III. **The Quantum Theory of Damping for Scattering Processes of Particles and Radiation.** W. PAULI, *Institute for Advanced Study.*

CONTRIBUTED PAPERS

11. Infra-Red Spectroscopic Study of the Hydrogenation of Unsaturated Compounds. ROBERT C. GORE AND JAMES L. JOHNSON, *American Cyanamid Company.*—Several absorption bands in the infra-red region between 1000 and 800 cm^{-1} have been correlated with mono-, di-, and tri-alkyl substituted $\begin{vmatrix} & \\ -C=C- \\ & \end{vmatrix}$ groups.* Generally a double bond is reactive to halogen addition. In cases where addition of halogen can be accomplished without destroying the molecule or without too greatly confusing its spectrum, this addition offers a method for checking some of these assignments, and also for investigating certain unsaturated molecules failing to show the 1640 cm^{-1} C=C stretching vibration which is an accepted criterion for unsaturation. In these cases the disappearance of bands at frequencies absorbed by the original molecule may be indicative of the presence of a $\begin{vmatrix} & \\ -C=C- \\ & \end{vmatrix}$ group. One of the more interesting of

these assignments is that of the 830 cm^{-1} band in crepe rubber to the tri-alkyl olefin.

$$\begin{array}{c} CH_3 \\ | \\ -CH_2-C=CH-CH_2 \end{array}$$

When Wijs reagent is used to add halogen across the double bond the 830 cm^{-1} band does not appear in the product. Similarly halogenation causes the disappearance of 990 and 910 cm^{-1} bands attributed to the group $RCH=CH_2$, and the 960 cm^{-1} band associated with the structure $R_1-CH=CHR_2$. Smaller molecules are treated by bromine addition and their spectra studied in the same manner. This procedure has made possible studies on bands attributed to the various $\begin{array}{c} | \\ C=C \\ | \end{array}$ groups mentioned above; the method can also be extended to aryl constituents and cyclic compounds where double bond characteristics are less obvious spectroscopically.

* Thompson and Torkington, Trans. Faraday Soc. 41 246–260 (1945).

12. Ultrasonic Interference at Angular Reflection. G. W. WILLARD, *Bell Telephone Laboratories.*—When a plane-wave ultrasonic beam is reflected at a plane surface the incident and reflected waves may interfere in the region of their coexistence and produce standing waves parallel to the reflector. The spacing of the standing waves, $h=\lambda s/2 \sin\theta$ (λs is the sound wave-length and θ is the angle between beam and reflector), increases rapidly with decreasing angle from $\lambda s/2$ at normal incidence to $h=\lambda s/2\theta \doteq 29\lambda s/\theta^\circ$ at grazing incidence. With a 10-megacycle sound beam in water ($\lambda s=0.15$ mm) the standing waves, as well as the incident and reflected beams, may be imaged on a screen by the ultrasonic light diffraction method, using a parallel light beam through the liquid, pinhole entrance and annular exit apertures (giving a dark field), and a projecting lens beyond the exist aperture. At all angles, from grazing incidence up to normal incidence, the standing waves produce on the screen simple bright bands parallel to the brass reflector. Magnification of the image is necessary except near grazing incidence.

13. The Absorption of Microwaves by Gases. W. D. HERSHBERGER, *RCA Laboratories, Princeton, N. J.*—The absorption by ammonia of electromagnetic waves having a length in the one-centimeter range has been known for some years. An investigation has been made to determine whether other gases show similar absorption for microwaves and as a result fourteen additional gases have been found whose absorption is comparable to that of ammonia. Among these gases are dimethyl ether, a variety of amines and alkyl halides, and several others. Measurements on the absorption coefficient and dielectric constant of these gases are given. The frequency at which the absorption coefficient attains its maximum value is inferred from the curve: absorption coefficients *vs.* pressure. Data on the absorption of several gas mixtures are given. Possible molecular mechanisms adequate to account for the large absorptions observed are discussed together with the conclusions reached.

14. Experiments with Triode Counters. S. A. KORFF, *New York University.*—Further studies of the properties of triode counters have been carried out. Various grid-diameters have been tried. Small diameter grids are found to produce considerable lowering of the over-all operating potential, but also introduce a constructional problem which is readily solved for short counters, but involves mechanical difficulties in the longer types currently used in cosmic ray studies. The application of grid counters to the problem of proportional counting is complicated by the collection of ions on the grid, which introduces uncertainties into the measurement of pulse sizes. The potential difference between the grid and cylinder is found not to be at all critical, and can be varied over wide limits without making appreciable differences in the operation of the counter. Resolving-time measurements show that the resolving-time of counters is made shorter by the introduction of a grid.

15. Reflex Oscillators Utilizing Secondary Emission Current. C. C. WANG, *Westinghouse Electric Corporation.*—One type of electronic oscillator, widely known as a velocity modulation oscillator or klystron uses an electron beam traversing the gap of a reentrant cavity resonator or several gaps in succession. The linear dimensions of such resonator must be made nearly in direct proportion to the wave-length of the electric oscillation to be generated. As the density of an electron beam that can be generated from a thermionic cathode cannot be increased indefinitely, the electron current that can be carried through the gap opening of the resonator will decrease nearly in proportion to the square of the wave-length. On the other hand, the cavity losses relative to the electrical field strength across the gap increase as the wave-length decreases. Consequently, for very short wave-length, it is very difficult to obtain appreciable power output from such tubes. By introducing a secondary emission electrode in the tube at a proper point, much higher beam current can be delivered to the gap where high frequency energy is absorbed from the beam. This results in a substantial increase of efficiency and available useful power output. Since such oscillators have been successfully made to operate as high as 4000 mc, it is apparent that either the average time lag of emission of secondary electrons is small in comparison with one cycle of oscillation or $1/4\times10^{-9}$ sec., or the dispersion of time of lag of emission of the secondary electrons is not very extensive if the average time of lag is not small.

16. Electronic Spectroscopy. G. C. SZIKLAI AND A. C. SCHROEDER, *RCA Laboratories, Princeton, N. J.*—According to Einstein, the electrons leave a photoelectric surface with a velocity proportional to the frequency of the impinging radiation. When a periodically variable potential is applied to a photo-cell, current flow starts at the threshold potential, gradually increasing to the saturation current. The linearity of this current depends on the linearity of the variable potential, the photoelectric surface, and the color content of the light. By double differentiation, the spectral distribution can be directly observed on an oscilloscope which is deflected synchronously with the potential

applied to the photo-cell. With caesium-antimony surfaces, filters of the three primary colors can be easily differentiated, but with improved surfaces considerably better resolutions may be obtained and further improvement may be had by lowering the temperature of the photosurface. The method lends itself readily to color matching, since two similar devices supplying their signal with opposite polarity will give zero output when the two colors match, while it provides a direct indication of the color and magnitude of mismatch when a signal is obtained. The output signal can be utilized for relays or other control devices.

17. A Voltage Regulator for X-Ray Circuits. WHEELER P. DAVEY, *The Pennsylvania State College.*—Power for operating our x-ray tubes is obtained from a 20 kva 110-V 60-cycle alternator. Experiments involving x-ray tubes frequently require a very constant voltage supply of considerable power. Commercially available voltage regulators are not sufficiently sensitive for the purpose. A highly statisfactory type of voltage control can be made as follows. By means of an ordinary four-tube rectifier the 110-volt a.c. line circuit is rectified. Voltage control is obtained in terms of the peak of the wave by connecting a 100 mf condenser across the rectified potential. The rectified potential is then connected across two Ohmite potentiometers in series (a total of 1500 ohms), thus making it possible to pick off definite fractions of the d.c. voltage. The adjustable voltage thus obtained is connected in series with a bucking voltage obtained from dry cells. In series with the dry cells is a Weston Instrument Co. d.c. relay model 534 which requires an operating current of 15 microamperes. The contacts of these sensitive relays operate two Weston Instrument Co. relays model 712, type 4, whose coils operate on 6 volts d.c. with contacts rated at 5 amperes on 110-volt d.c. These relays are especially designed to work with a coil current small enough to be safe for the contacts of the 534 relay. The second relays are in turn connected to two Struthers Dunn mercury contact relays type 22 BXX with double contacts normally open. These mercury relays together act like a double-pole double-throw switch. They actuate the armature of a 1/6 hp d.c. motor. When the rectified line voltage is too high the motor therefore operates in one direction and when the rectified line voltage is too low the motor operates in the opposite direction. The shaft of the motor is connected to Cenco resistances which serve as the field rheostat of our 110-volt 60-cycle alternator. A voltage regulator such as has been described above has been in constant use in our x-ray laboratory for twelve months. It regulates the a.c. voltage to within $\pm 1/50$ of a volt except for the short time intervals required for the motor to set the slider of the rheostats.

18. Secular Magnetic Variations as Transients. WALTER M. ELSASSER, *Columbia University.*—The quadrupole and higher harmonic components of the earth's magnetic field are subject to large secular variations within intervals of time of the order of a few hundred years. Observation indicates that the non-dipole part of the field has very nearly a completely random distribution in time. Its Fourier spectrum consists therefore in the main of components with periods of the order of a few centuries, the constant components being small or zero. The inductance of the earth's metallic core (assuming a fair metallic conductivity) is very large; as a result the periods of spontaneous decay of electric currents in the core, computed by conventional methods, are of the order of 10^4–10^5 years. Under these conditions it can be shown that the electric currents causing the non-dipole part of the field must flow in the top strata of the core. In the case of variable currents flowing at greater depth the upper part of the core shields the outer space from all "short period" Fourier components. The location ascribed here to the currents agrees with earlier results.[1]

[1] Phys. Rev. 60, 876 (1941).

19. A New Magnetomotive Force Gauge and Magnetic Field Indicator. W. B. ELLWOOD, *Bell Telephone Laboratories.*—A new gauge and method of measurement are described, whereby the distribution of magnetic potential may be determined along the ferromagnetic core of a telephone relay, or similar electromagnetic device, while it is mounted in operating position, without disassembly, special search coils, or even change in adjustment. The gauge is portable, compact, and sensitive. A glass-enclosed, magnetically-operated reed switch, which is used because of its small size, high sensitivity, and magnetic structural features, is held electrically closed by the magnetomotive force which it is desired to measure. This magnetomotive force is opposed by a countermagnetomotive force supplied by a winding on the switch. The current in the winding which opens the switch is the measure of the magnetomotive force. The necessary control circuits and the procedures for their use are described for both manual and automatic operation. The device and method are applicable to the study of permanent magnets as well as electromagnetic structures. The coercive force of the core of a relay may be determined in a nondestructive manner.

20. The Movement of Solid Bodies in Sunlight. FELIX EHRENHAFT, *New York City.*—Experimental proof has been given that in all question of matter and radiation the unipolar north and south magnetic charge has to be taken into account in the same manner as the unipolar positive and negative electric charge.[1] Because of the great intrinsic intensity photomicrographs of microscopic bodies have been obtained during their helical light positive or light negative photophoretic movement in the concentrated beam of sunlight.[2] Photomicrographs permitting quantitative evaluation will be shown. In air of atmospheric pressure smoke particles, for instance, make 125 to 150 regular spaced turns per second. Light positive or light negative copper particles make about 20 turns per second, the helical traces are about 10^{-3} cm in diameter or 20 times larger than the radius of the copper particles. A homogeneous constant magnetic field of 50 gauss parallel to the beam of sunlight was applied half a second on an iron particle during its movement. The diameter of the helical path is about 100 times larger than the radius of the particle when light and the external magnetic field are acting together. The

diameter of the helical path in sunlight alone is much smaller. From the light negative photophoretic velocity in air of atmospheric pressure and the helical traces described above, with and without external magnetic field, the longitudinal constant magnetic field[3] in sunlight and the electric and magnetic charge of the particle has been calculated.

[1] F. Ehrenhaft, Bull. Am. Phys. Soc. Stanford Meeting, July, 1945.
[2] F. Ehrenhaft, Bull. Am. Phys. Soc. Columbus, Ohio Meeting, June, 1945.
[3] F. Ehrenhaft, Science 101 676 (1945).

21. Law of Equilibrium Between Two Liquids. GEORGE ANTONOFF, *Fordham University*.—Gibbs formulated a mathematical law that at the boundary of two media the tension equals the difference of surface tensions of both. Actually for the liquid phases in equilibrium a law of the same form, but with different numerical values holds true. It can be deduced theoretically subject to condition that both phases contain an equal number of moles per unit volume. In fact, both have identical colligative properties, the reason why they don't mix. Separation into two liquid phases can be regarded as a condensation of a substance within a solvent. It undergoes polymerization resulting in more than one molecular species. Equilibrium depends on a reversible reaction which is time dependent. Thus, physical properties, among them densities, exhibit considerable and sometimes rapid fluctuation which cease at equilibrium. Thus, deviations from the above law cannot be interpreted as surface phenomenon, being due to processes in the bulk of solutions. Systems liquid-liquid present many experimental advantages as compared with other systems, and through them a comprehensive theory of liquids can be formed, which is in perfect agreement with experimental evidence. The theory of liquids is handicapped by want of suitable methods, it can only find solution if one falls back on ideas of Gibbs on equilibria.

22. An Equation of State for a System with a Single Type of Transformation, Including the Metastable States. J. L. FINCK, *The J. L. Finck Laboratories, Brooklyn 25, New York*.—Metastable states (supercooled and superheated fluids, etc.) have not been included among the equilibrium states considered in thermodynamics because such states are considered irreversible. Concept of reversibility is considered, and it is found to depend upon number of independent variables of system. For single homogeneous phase subject only to mechanical and thermal changes, just two independent variables have been considered. This practice requires that some states be characterized as metastable. Justification is offerred for change in this accepted view. Instead of classifying systems in phases, it is proposed to consider systems on basis of number of types of transformation. Single gaseous phase with dissociation will be analogous to liquid-vapor system. Each of these systems will have three independent variables, such as p,v,T, and equations of state will be of form $\epsilon = F(p,v,T) = G(p,T,x)$, and $v = (p,T,x)$ where ϵ is internal energy and x degree of transformation. These equations include all metastable states. On this basis an explicit equation of state for system with single type of transformation is developed, and results in the form

$$h = A + Bv + Cp + Dpv - T(E + Fv + Gp + Hpv)$$

where h = total heat, A, B etc. constants. This equation checks with data on saturated and superheated ammonia to within a few tenths percent. Also, for saturated and superheated steam agreement with steam tables is within several percent over the entire range of tables.

23. The Distribution of Energy in Turbulence. LARS ONSAGER, *Yale University, New Haven, Connecticut*.—The dissipation of energy by turbulence is regarded as primarily a "violet catastrophe." The velocity field of a liquid has an infinite set of Fourier components, whose mutual modulation redistributes the energy among more and more components which belong to ever increasing wave-numbers. In actual liquids this subdivision of energy is intercepted by the action of viscosity, which destroys the energy more rapidly the greater the wave number. However, various experiments indicate that the viscosity has a negligible effect on the primary process; hence one may inquire about the laws of turbulent dissipation in an ideal fluid. The modulation of a given Fourier component of the motion is mostly due to those others which belong to wave numbers of comparable magnitude. Some important applications of this principle are known; but it has not been pointed out before that the subdivision of the energy must be a stepwise process, such that an n-fold increase of the wave number is reached by a number of steps of the order $\log n$. For such a cascade mechanism that part of the energy density which is associated with large wave numbers should depend on the total volume rate of dissipation Q only. Then dimensional considerations require that the energy per component of wave number k equal (universal factor) $Q^{2/3}k^{-11/3}$. The corresponding correlation-function for the velocities at two points r apart has the form $R(r) = 1 - (\text{const.})\, r^{2/3}$.

24. The Wave Equation in a Medium With a Variable Index of Refraction. PETER G. BERGMANN, *Columbia University (On leave of absence from Lehigh University)*.—Occasionally one is confronted with the problem of solving the wave equation when the index of refraction is a continuous function of the coordinates. It is well known that in this case the rigorous wave equation contains a first-order term in addition to the standard terms. This term is usually neglected. Its effect on the propagation of sound has been estimated by means of two different methods. The first-order term can be eliminated for any given frequency by introducing as the wave variable the excess pressure, divided by the square root of the density. The resulting equation contains a modified index of refraction which depends on the frequency. The second method, which is usually employed to examine the relationship between wave and ray treatment, involves the introduction of an amplitude and a phase factor. It is found that the first-order term appears in the "eikonal equation." It is proportional to the square of the wave-length, and it is small compared with the "diffraction term" unless the index of refraction changes appreciably over a distance of the same order as the radius of curvature of the wave front.

25. Space Charge—Limited Beams in Electrostatic Fields. M. E. ROSE,*[1] *Sonotone Corporation.*—The behavior in an electrostatic field of an electron beam limited by space charge is considered. The case dealt with is the circularly symmetric beam of finite cross section. Only first-order optics in which the aberration is due to space charge is treated. From the transformed equation of radial motion it is possible to obtain (a) the shape of the beam r for a given axial potential V, or (b) the axial potential for a given shape of beam. The electrode shapes are then to be obtained from the axial potential, or, *vice versa*, by integrating the Laplace equation. In general a numerical integration of the second-order equation of motion is necessary. However, by introducing $R=rV^{1/4}$ problem (b) may be reduced to a first-order, non-linear equation, which simplifies the procedure considerably, at least in some cases. The electrode design which will collimate an electron beam[2] can be obtained rigorously. The results obtained are in excellent agreement with the measurements carried out in the electrolytic tank.[3] In other cases it is possible to obtain the axial potential, and hence the beam radius, directly from the first-order equation. The problem of the "cross-over" in the presence of space charge is examined and it is concluded that a cross-over is not possible.

* To be read by title only.
[1] Now at Illinois Institute of Technology.
[2] J. R. Pierce, J. App. Phys. 11, 548 (1940).

26. The Ranges of Small Samples. RALPH HOYT BACON, *Fairchild Camera and Instrument Corporation, New York, N. Y.*—The range, or extreme variation, R_n, of a sample of n specimens, is the difference between the largest and the smallest specimen of the sample, and is the simplest statistic for describing the dispersion of the sample. There are applications for which it is of interest to know the distribution, $\psi(R_n)$, of the ranges of small samples drawn from a lot or universe whose distribution function, $\phi(x)$, is known. The distribution of the ranges is related to the parent distribution by the equation,

$$\psi(R_n)=n(n-1)\int_{-\infty}^{+\infty-R_n}\left[\int_x^{x+R_n}\phi(x)dx\right]^{n-2}\times\phi(x)\phi(x+R_n)dx.$$

Except for a few special cases, it is not possible to solve this equation explicitly, although the moments of $\psi(R_n)$ for samples of various sizes drawn from a normal universe were found by Tippett[1] in 1925. In the present work, the parent distribution is replaced by a histogram. Expressions for the number of samples of a given size, n, having a given range, R_n (measured in multiples of histogram intervals), obtainable from the distribution represented by the histogram are easily derived. In the limit (infinite number of histogram intervals of infinitesmal width), these expressions yield series expansions of $\psi(R_n)$, where the coefficients of the terms of the series are functions of certain definite integrals related to the parent distribution function, $\phi(x)$, and of the sample size, n.

[1] L. H. C. Tippett, Biometrika 17, 364 (1925).

27. Relativistic Interaction of Electrons on Podolsky's Generalized Quantum Electrodynamics. D. J. MONTGOMERY, *Princeton University.*—Podolsky[1] has formulated the basis for a generalized electrodynamics involving higher derivatives in the field equations. Podolsky and Kikuchi[2] have developed the theory to include the quantum case, getting natural generalizations of ordinary quantum electrodynamics, and have shown the self-energy and particle-particle interaction to be finite. The present paper extends the formalism, basing the treatment on some work of Fock,[3] and applies the results to the determination of the relativistic interaction of two electrons. In order to obtain an expression for this interaction, the wave equation for a system of particles is first derived, and the wave functional is developed in a series of eigenfunctionals. The resulting equations are transformed into a modification suitable for perturbation treatment, with the unperturbed system taken as two free electrons. The matrix element for the interaction turns out to be a generalization of Møller's formula.

[1] B. Podolsky, Phys. Rev. 62, 68 (1942).
[2] B. Podolsky and C. Kikuchi, Phys. Rev. 65, 228 (1944); 67, 184 (1945).
[3] V. Fock, Physik. Zeits. Sowjetunion 6, 425 (1934).

28. On the Heavy-Electron Pair Theory in the Stong Coupling Limit. JOHN M. BLATT, *Princeton University.**—The pair-theory of Marshak and Weisskopf[1] was investigated assuming strong coupling. The strong-coupling criterion is $A=(Nf/\mu)>5$ where f=coupling constant, μ=heavy electron mass, $N=\int U^2(x)d^3x$, $U(x)$=source function of the nucleon. $N\cong 1/\pi a^3$, where a=source radius (all in units where $\hbar=c=1$). With this condition, the magnetic moment turns out to be of order A^{-1}, and of magnitude too small to account for the observed anomalies. The leading term in the potential of the force between two nucleons (for $r_{AB}>2a$) is independent of the spin orientations, of the coupling constants, and of the type of coupling (as long as no derivatives of the heavy-electron field quantities occur in the coupling term). This potential is identical with the one calculated by Jauch[2] and Houriet,[3] but was not considered by Nelson and Oppenheimer.[4] The next term in the potential is of order A^{-2}. It corresponds to a superposition of a $(\Sigma_A\cdot\Sigma_B)$ term, a tensor force term, and an ordinary force. Being of order A^{-2} it is too small, however, to fit the experimental results.

* Now with RCA Laboratories, Princeton, N. J.
[1] Marshak, Phys. Rev. 57, 1101 (1940); Marshak and Weisskopf, Phys. Rev. 59, 130 (1941).
[2] Jauch, Helv. Phys. Acta 15, 175 (1942).
[3] Houriet, Helv. Phys. Acta 16, 529 (1943).
[4] Nelson and Oppenheimer, Phys. Rev. 61, 202 (1942).

29. Equilibria in Pure Liquids.* GEORGE ANTONOFF, *Fordham University.*—Condensation of vapors into a liquid follows the pattern of systems separating into two liquid layers, with the difference that the solvent is absent, and it takes place so to say in vacuum. Thus, the vapor and liquid in equilibrium are subject to the same laws as systems liquid-liquid. Properties of many liquids have been found time dependent. This is due to the presence of several molecular species. Equilibrium between them is

sometimes reached rapidly, as in the case of ionic substances, e.g. water. In other cases the process is slow. This explains why physical constants of liquids by different authors often vary withing limits exceeding experimental errors.

* To be read by title only.

30. On Position of Uranium Y in Radioactive Series.* GEORGE ANTONOFF, *Fordham University.*—Since I isolated a branch product in uranium which I called uranium Y, various hypothesis were expressed as to its position in the series. It is sometimes regarded as a parent of protactium, in which case the uranium atom would disintegrate at a rate of 98 percent in the main series, and about 2 percent in the direction of the actinium series. There was also expressed the opinion that an element of atomic number 92 and atomic weight 235 may be the parent of uranium Y. This hypothesis is in better agreement with facts known, if one assumes that U^{235} is normally present in uranium in quantity of about 0.7 percent. The amount of uranium Y I ever succeeded in separating never exceeded this amount. The hypothesis that uranium Y may be connected with uranium Z, a branch product of uranium X_1 to the extent of 0.035 percent of the latter, is therefore not supported by experimental evidence, because this figure is of a different order of magnitude.

* To be read by title only.

Proceedings of the American Physical Society

MEETING OF THE NEW YORK STATE SECTION OF THE AMERICAN PHYSICAL SOCIETY AT BUFFALO, NEW YORK, NOVEMBER 10, 1945

THE Fall Meeting of the New York State Section of the American Physical Society was held in Foster Hall, University of Buffalo, Buffalo, New York on Saturday, November 10, 1945. This meeting was sponsored jointly by the Section and the New York State Science Teachers Association, and marks the return of the Section to an active status, the Spring meeting having been cancelled because of the limitations on attendance imposed by the Office of Defense Transportation. An attendance of about fifty persons was recorded at the sessions.

The following program of invited papers was presented:

Morning Session

Address of Welcome, SAMUEL P. CAPEN, *Chancellor, The University of Buffalo.*

Focusing Invisible Light, RAYMOND T. BYRNE, *Batavia High School.*

Problems of the Spectrochemical Analysis of Metals, T. S. JACKSON, *The Bethlehem Steel Company.*

Applications of the Mass Spectrograph, D. L. WOERNLEY, *The University of Buffalo.*

Afternoon Session

Properties of a Sensitive Point Counter, AUSTIN C. MCTIGUE, *Canisius College.*

Some Diffraction Problems in Phase Difference Microscopy, HELEN JUPNIK, *Spencer Lens Company.*

The Wind Tunnel and Airplane Design, F. W. GEIGER, *Curtiss-Wright Corporation.*

W. R. FREDRICKSON
Secretary

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